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Significance of the Secant and Tangent Moduli of Elasticity in Structural Design

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Introduction

THIS PAPER deals with the problem of designing beams and columns of structural materials that have no well-defined yield point and which must be used above their proportional limits to be useful.

Other investigators have demonstrated that analysis of such structures can be made by means of the concept of the reduced modulus of elasticity. The reduced moduli for beams and columns of solid rectangular sections have been developed and checked experimentally. The author develops formulas for determining reduced moduli of hollow sections and shows that columns can be properly designed by direct use of tangent moduli in Euler's formula. Confirmatory tests were made on columns of corrugated stainless steel. The directional properties of these stainless-steel columns are shown by the experimental data to have a relatively small effect on the column performance. Likewise, stress-relieving for general improvement of physical properties did not appear to produce commensurate load-bearing capacity in these columns.

It is shown that the reduced modulus for beams of hollow section is substantially equal to the secant modulus.

Theory of Use of the Tangent Modulus

The theory of flexure beyond the proportional limit has come into use in aircraft and other lightweight structures rather recently but is not generally known or applied outside of these circles. The classic formulas for strength and deflection of beams and columns assume a constant modulus of elasticity throughout the member, whether the stress be tension or compression, even when the stress level is carried to the yield point of the material.[1] Engineers have recognized basic differences in the initial moduli of elasticity of materials such as steel or iron, brass, and aluminum. These initial moduli always prevail to the proportional limit, and in the strain-free iron and steel this is practically the yield point. In materials such as brass and aluminum, no sharp yield usually exists. There is a more or less rapid decrease in the modulus of elasticity as the stress is increased above the proportional limit.

In materials such as austenitic stainless steel, a sharp yield point seldom, if ever, occurs, and ordinarily the proportional limit is considerably below its yield strength. When austenitic stainless steel is cold-reduced, the proportional limit, yield, and tensile strengths can be increased.[2, 3]

The ferritic steels lose their sharp yield points when subjected to small amounts of strain (1 or 2 per cent), a principle that is put to use in the form of temper rolling to eliminate "stretcher-strains," which are so objectionable in drawn objects such as autobody parts. The low-alloy, low-carbon, high-strength steels being developed for aircraft owe their strength to cold reduction and exhibit stress-strain curves similar to those for cold-rolled austenitic stainless steels. It is evident that some of the knowledge gained in the past on stainless steels will be useful in the application of these new steels in aircraft.

The suitability of materials for structural purposes has been judged mostly by the tensile properties, largely because of convenience and precedent. This practice is satisfactory as long as the material is structurally isotropic. However, the demand for lightweight structures is causing the increased use of cold-reduced steels in which the tensile and compressive properties are often unequal and also variable relative to the rolling direction of the material.

The Army-Navy Aeronautical Specification AN-QQ-S-772 classifies aircraft stainless steel of the 18–8 chromium-nickel type according to its yield and tensile strengths. Material from which the tensile speci-

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mens give yield strengths exceeding 110,000 lbs. per sq.in. (0.2 per cent offset) and ultimate strengths exceeding 150,000 lbs. per sq.in. is called "half-hard"; material having yield strengths exceeding 140,000 lbs. per sq.in. and tensile strengths exceeding 185,000 lbs. per sq.in. is called "full-hard." It must be noted, however, that the compression yield strength along the grain of the half-hard stainless steel sometimes tests as low as 85,000 lbs. per sq.in.[4] The design of lightweight structures is mainly concerned with the strength of compression members, which are inherently less strong than tension members due to (1) secondary failure by local instability of thin sections and (2) primary column failure in general.

The behavior of compression members, struts, or columns was first rationalized by L. Euler almost 200 years ago.

End conditions in columns have been handled in many standard textbooks[5, 6, 7] and are mainly important insofar as they determine the number and nature of counterflexures leading to primary failure.

Investigators and engineers of the last century found Euler's formula to be adequate for long columns made of sharp-yielding materials but not for short columns (L/r less than 100), and as a consequence the empirical formulas of Rankine, Gordon, Johnson, and others were worked out to enable the engineer to determine permissible column loads in the short-column range.[8, 9]

Unrestrained columns of sharp-yielding material theoretically do not develop critical stresses greater than the yield point, and experiment bears out this fact.[10]

Safe and efficient long columns (L/r more than 100) can be designed by means of Euler's formula, provided the number and nature of counter-flexures occurring under load are properly taken into account, because a material like ordinary mild steel having a modulus of elasticity of 30,000,000 lbs. per sq.in. would be stressed to no greater than 30,000 lbs. per sq.in. at a slenderness ratio of 100:1. This would ordinarily not exceed the proportional limit and, consequently, the modulus of elasticity would be constant up to this stress.

With slenderness ratios less than 100:1 Euler's equation indicates the development of stresses greater than the yield point in ordinary mild steel. Failure under such conditions is therefore not due to primary Euler buckling. Relations like the Johnson[11, 8] parabolic formula for short columns have been successfully applied to sharply yielding materials because it is relatively easy to fit the Johnson parabolic relation to limiting cases—namely: (1) At a slenderness ratio of zero, the terminus cannot exceed the yield point; (2) Johnson's relation is made tangent to Euler's.

Inasmuch as austenitic stainless and cold-reduced steels do not exhibit sharp yield points, a yield strength based on the stress at 0.2 per cent offset strain is used as a measure of useful strength. There is no fundamental relation between this arbitrary yield strength and short-column failure. Correlative tests between yield strength and short-column failure must be made so that short-column formulas such as Johnson's parabolic expression can be fitted to the resulting data and used generally for other short columns of the same material.

The defect in Euler's relation when applied to short columns appears to be due mainly to the modulus of elasticity changing from the initial value used in the equation. Osgood[12] summarizes the work of Considère (1889) and others by saying that "as an ideal column stressed beyond the proportional limit begins to bend, the stress on the concave side increases according to the law of the compressive stress-strain diagram and the stress on the convex side decreases according to Hooke's law, and that therefore the strength would be given by

$$P = \pi^2 EI/L^2 \tag{1}$$

in which $\overline{E}$ is a modulus, the value of which lies between the modulus of elasticity and the tangent modulus. . . . Earlier in 1889 Fr. Engesser presented his tangent modulus, and in 1895 Félix Jasinski pointed out that this theory was not correct and called attention to Considère's work. . . . . Kármán (1910) presented the theory again, adding the actual evaluation of $\overline{E}$ for the rectangular cross section and the idealized H-section (consisting of infinitely thin flanges and negligible web), and gave the theory new life by making a series of careful tests designed to afford a check on the theory. Since then $\overline{E}$ has been evaluated for other cross sections by a number of writers."

The double modulus or reduced modulus, as Timoshenko calls it, for the solid rectangular section is usually given in the following form:[13, 14]

$$E_r = 4E_tE_0/(\sqrt{E_0} + \sqrt{E_t})^2 \tag{2}$$

where

E_r = reduced modulus of elasticity, lbs. per sq.in.
E_t = tangent modulus of elasticity, lbs. per sq. in.
E_0 = initial or Young's modulus of elasticity, lbs. per sq.in.

Most of the sections used in aircraft are substantially hollow or made to dispose the material as remotely from the neutral axis as practical in order to obtain the maximum strength and stiffness with the minimum of material. Box beams, monocoque structures, and corrugated sections are examples of this practice. The reduced modulus of elasticity for hollow sections in which the material is ideally remote from the neutral axis will now be derived.

Let the material have a stress-strain curve as shown in Fig. 1, the stress-strain relation being defined by the curve OA when the stress is increased from zero to the maximum value F, at which point the tangent shown is a measure of the tangent modulus. If the point A on the stress-strain curve corresponds to the start of primary buckling of the hollow column shown in Fig. 2,

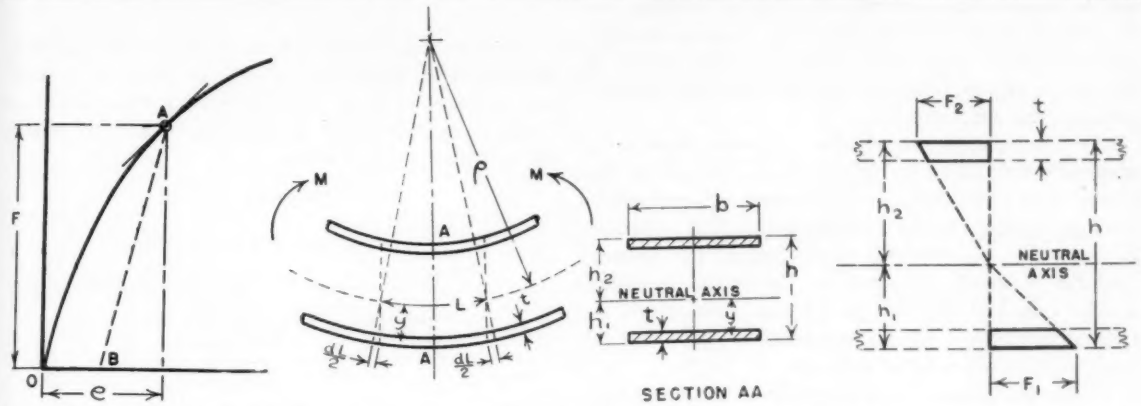


FIG. 1 (*left*). Idealized stress-strain curve. FIG. 2 (*middle*). Schematic diagram of a bent hollow rectangular section. FIG. 3 (*right*). Stress distribution in hollow rectangular section acting as a column.

the configuration of stress in the material will be as represented in Fig. 3.

The values of stress F_1 and F_2 represent the difference between the fiber stress throughout the column and the average stress over the whole section. This difference is due to bending. The stress on the concave side of the column will lie on line OA of Fig. 1, while the stress on the convex side will be represented approximately by line AB.

To simplify this derivation, assume that the thickness t is extremely small and that the distance h is relatively great. By the same reasoning let it be assumed that the stresses F_1 and F_2 are substantially constant throughout the thickness in each case. For a position of stable equilibrium in the slightly bent column the forces on the concave and convex sides of the section must be equal. The position of the neutral axis must be such as to bring about this balance. Basic mechanics teaches that the stress in a section subjected to a moment M is

$$F = Mc/I \tag{3}$$

where I = moment of inertia, in.^{4}, and c = extreme fiber distance, in. Likewise, the relation of this moment M to the radius of curvature is

$$M = EI/\rho \tag{4}$$

where E = appropriate modulus of elasticity, lbs. per sq.in., and ρ = radius of curvature, in. By substitution

$$F = Ec/\rho \tag{5}$$

The total forces must be equal to each other for equilibrium. Using the appropriate moduli,

$$(bt)(E_t h_2/\rho) = (bt)(E_0 h_1/\rho) \tag{6}$$

or simplifying

$$E_t h_2 = E_0 h_1 \tag{7}$$

but

$$h = h_1 + h_2 \tag{8}$$

therefore

$$h_1 = E_t h/(E_0 + E_t) \tag{9}$$

Similarly

$$h_2 = E_0 h/(E_0 + E_t) \tag{10}$$

The total moment of forces acting about the neutral axis is not necessarily zero,[15] but rather

$$M = \left[\frac{E_0 h_1 (bt) h_1}{\rho}\right] + \left[\frac{E_t h_2 (bt) h_2}{\rho}\right] \tag{11}$$

whence

$$M = (bth^2/\rho)[E_0 E_t/(E_t + E_0)] \tag{12}$$

But

$$I = 2(h/2)^2 tb = h^2 tb/2 \tag{13}$$

so

$$M = (I/\rho)[2E_0 E_t/(E_t + E_0)] \tag{14}$$

Let

$$E_r = 2E_0 E_t/(E_t + E_0) \tag{15}$$

E_r will then be a reduced modulus, in this case for hollow rectangular sections.

$$M = IE_r/\rho \tag{16}$$

When this equation is manipulated as the original Euler derivation it takes the classic Euler form as follows:

$$F = P/A = \pi^2 E_r/(L/r)^2 \tag{17}$$

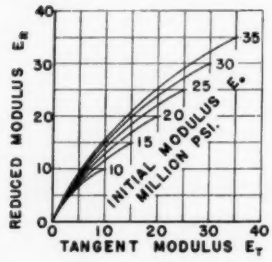
This equation can be used to design short columns at stresses exceeding the proportional limit.

The solid rectangular and hollow rectangular sections with corresponding reduced moduli given in Eqs. (2) and (15), respectively, probably represent the extreme conditions in useful sections likely to be encountered. Admittedly, it would be a tremendous

task to prepare data for reduced moduli of elasticity for all structural shapes that might be used in aircraft, especially since there are so many of them. Some of the most useful materials are anisotropic, which further increases the difficulties.

The next question is whether a simpler index of a material's suitability for short columns might be devised. The initial modulus of elasticity E_0 appears in the Eqs. (2) and (15) for the solid and hollow rectangular sections. In the high stress range, the tangent modulus is small compared to the initial modulus, which varies little for a given material. Therefore, the tangent modulus is the major quantity influencing the magnitude of the reduced modulus.

The curves in Figs. 4 and 5 have been calculated from Eqs. (2) and (15). When applied to hollow rectangular sections of steel used in short columns, an initial modulus of 30,000,000 lbs. per sq.in. might be assumed. If the stress level were such as to cause the material to be acting along the stress-strain curve at a tangent modulus of 10,000,000 lbs. per sq.in., the reduced modulus would be 15,000,000 lbs. per sq.in.; under the same conditions in a solid rectangular section the reduced modulus would be about 15,800,000 lbs. per sq.in.



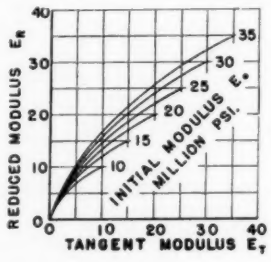


FIG. 4 (*left*). Reduced modulus as a function of initial and tangent moduli in hollow rectangular sections. FIG. 5 (*right*). Reduced modulus as a function of initial and tangent moduli in solid rectangular sections.

These differences are even less significant in short columns than would appear. The critical column stress, F, determines the tangent modulus effective at the point where primary buckling occurs. When values for E_T and F are substituted in Eq. (17) the corresponding slenderness ratio can be determined. However, for a constant critical stress, a given change in the tangent modulus corresponds to a somewhat smaller change in the slenderness ratio, which term is in the second degree. Conversely, small changes in the slenderness ratio in the short-column range must be offset by somewhat larger changes in the tangent modulus, provided the same critical stress level is required. This means that the decrease of the tangent modulus at the high stress levels is not so significant in columns as might be expected. This principle is difficult to show analytically; therefore column tests were made to demonstrate it experimentally.

MATERIAL

The material selected for the experiments was from two lots of the grade known as Type 301 or 17–7 high-tensile stainless steel. The American Iron and Steel Institute specifies that its analysis should be within the ranges given in Table I.

TABLE I

Analysis of Type 301 Stainless Steel	
Carbon	over 0.08% to 0.20% max.
Manganese	2.00% max.
Chromium	16% min. 18% max.
Nickel	6% min. 8% max.
Phosphorus	0.04% max.
Sulphur	0.04% max.
Silicon	1.00% max.

One of the lots, which will be referred to in this report as material A, met all of the requirements of Army-Navy Specification AN-QQ-S-772 for full-hard temper. The other lot, which will be called material B, did not quite meet the specified strength requirements for half-hard temper, although it is considerably stronger than would be necessary for quarter-hard temper. Both were approximately 25 gauge (0.022 in.) and were subjected to complete tensile and compression tests. Specimens were taken both longitudinal and transverse to the original rolling direction and tested as-cold-rolled and also after they had been stress relieved at 300°C. for 24 hours.[16]

Tensile tests were made in accordance with standards of A.S.T.M.,[17] using a 20,000-lb. Amsler universal testing machine. Compression tests were conducted in accordance with the method outlined by Franks and Binder.[3] The cylinder used by them is rolled up from flat stock and has a slenderness ratio not exceeding 15 and a ratio of diameter of cylinder to thickness of sheet not exceeding 40. The setup for the compression tests in the 200,000-lb. Baldwin-Southwark universal testing machine used in these experiments is shown in Fig. 6.

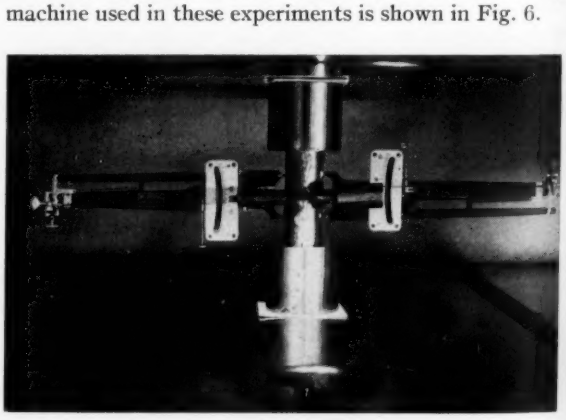


FIG. 6. Compression testing setup in 200,000-lb. Baldwin-Southwark testing machine.

TABLE II

MECHANICAL PROPERTIES OF 25 GAGE TYPE 301 (17-7)
HIGH TENSILE STAINLESS STEEL SPECIMENS

		DIRECTION OF SPECIMEN			
		Longitudinal		Transverse	
		As Cold Rolled	As Stress Relieved	As Cold Rolled	As Stress Relieved
TENSION			MATERIAL A		
Yield Strength at 0.2% offset - thousands of lbs. per sq.in.	Fty	160	170	154	170
Ultimate Strength - thousands of lbs. per sq.in.	Ftu	209	205	212	207
Proportional limit at 0.01% offset - thousands of lbs. per sq.in.	Ftp	83	92	86	106
Initial Modulus of Elasticity - Millions of lbs. per sq.in.	Eo	26.4	28.0	29.0	30.8
Elongation - Per Cent in 2 in.		18	16	11.5	9
COMPRESSION					
Yield Strength at 0.2% offset - thousands of lbs. per sq.in.	Fcy	133	161	161	191
Ultimate buckling strength - thousands of lbs. per sq.in.	Fcu	180	187	204	211
Proportional limit at 0.01% offset - thousands of lbs. per sq.in.	Fpy	43	97	98	120
Initial Modulus of Elasticity - millions of lbs. per sq. in.	Eo	25.2	25.6	29.1	28.4
TENSION			MATERIAL B		
Yield Strength	Fty	105	113	107	118
Ultimate Strength	Ftu	152	147	154	151
Proportional Limit	Ftp	54	56	54	67
Modulus of Elasticity	Eo	26.0	28.1	30.0	30.5
Elongation		27	33	26	23.5
COMPRESSION					
Yield Strength	Fcy	89	103	119	120
Ultimate Buckling Strength	Fcu	122	126	143	124
Proportional Limit	Fpy	41	59	68	70
Modulus of Elasticity	Eo	25.0	28.4	26.4	28.2

TABLE III

DIMENSIONS OF CORRUGATED SPECIMENS AND TEST VALUES

	MATERIAL B				MATERIAL A			
	Longitudinal		Transverse		Longitudinal		Transverse	
	As Cold Rolled	As Stress Relieved	As Cold Rolled	As Stress Relieved	As Cold Rolled	As Stress Relieved	As Cold Rolled	As Stress Relieved
				Nominal Length - 21 in.				
(1) Pitch "P", in.	1.37	1.39	1.34	1.35	1.35	1.36	1.39	1.39
(2) Depth "D", in.	.232	.212	.250	.231	.210	.201	.212	.206
(3) Pitch-Depth, "K"	5.90	6.56	5.36	5.85	6.45	6.76	6.56	6.75
(4) Tangent angle, "a"	28.8	26.5	30.8	28.4	24.5	23.4	24.6	24.9
(5) Thickness "t", in.	.021	.0196	.0212	.0196	.023	.023	.022	.022
(6) Width "b", in.	13.44	13.62	13.38	13.70	13.68	13.68	13.75	13.80
(7) Blodgett's strength factor "C_6"	.134	.132	.134	.133	.126	.126	.127	.129
(8) Development factor "λ_6"	1.065	1.058	1.080	1.068	1.055	1.052	1.053	1.052
(9) Least radius of gyration "r"	.0822	.075	.088	.0815	.0785	.0696	.0735	.0721
(10) Specimen length, in.	20.63	20.62	20.95	20.75	20.58	20.68	20.9	21.0
(11) Effective column length "L", in.	21.23	21.22	21.55	21.35	21.18	21.28	21.5	21.6
(12) Slenderness ratio "L/r"	259	282	245	262	292	306	293	299
(13) Critical load from test, lb.	1165	1155	1470	1175	1060	1000	1155	1125
(14) Cross-sectional area, sq.in.	.300	.288	.306	.287	.332	.330	.319	.320
(15) Critical column stress	3880	3960	4800	4100	3190	3030	3620	3520
				Nominal Length - 10.5 in.				
(11) Effective column length "L", in.	10.80	10.79	11.36	11.07	10.87	10.76	11.2	11.52
(12) Slenderness ratio "L/r"	133	141	129	136	153	148	152	160
(13) Critical load from test, lb.	4400	3980	5050	4600	3870	3650	3940	3740
(14) Cross-sectional area, sq.in.	.302	.285	.306	.287	.337	.330	.319	.320
(15) Critical column stress	14,600	13,900	16,400	16,000	10,600	11,100	12,900	11,700
				Nominal Length - 5.0 in.				
(11) Effective column length "L", in.	5.46	5.5	5.64	5.61	5.49	5.49	5.62	5.56
(12) Slenderness ratio "L/r"	68	74	64	71	79	81	77	77
(13) Critical load from test, lb.	12,900	11,450	16,150	15,000	11,300	13,250	15,150	15,200
(14) Cross-sectional area, sq.in.	.308	.282	.306	.287	.326	.325	.314	.320
(15) Critical column stress	41,900	40,600	52,800	52,200	34,600	40,700	47,500	47,500
				Nominal Length - 2.25 in.				
(11) Effective column length "L", in.	2.86	-	2.89	2.27	2.27	2.27	2.28	2.28
(12) Slenderness ratio "L/r"	35	-	33	38	40	41	39	40
(13) Critical load from test, lb.	24,400	-	26,900	21,850	27,200	26,200	30,800	30,200
(14) Cross-sectional area, sq. in.	.308	-	.366	.287	.326	.325	.319	.320
(15) Critical column stress	79,300	-	88,000	76,000	82,000	79,400	95,600	96,000

Items (1), (2), (3), (4), and (5) were averaged from a representative number of readings.

The summary of mechanical properties on lots A and B is given in Table II. As has been pointed out by other investigators,[2, 3, 4, 16, 18] the longitudinal compression strength of highly cold-worked materials is less than the corresponding tensile strength, and the transverse compression strength is greater than the corresponding tensile strength. This is true of materials A and B. Stress-relieving reduces these differences somewhat. The highest initial moduli of elasticity (29,000,000 to 30,800,000 lbs. per sq.in.) were observed in transverse tensile specimens. When the parts made from sheets have their working axes along the original rolling direction, which has less favorable compression strength, the stress analyst should have a complete understanding of this to make the best use of the properties.

The stress-strain curves upon which the above-mentioned data were based are given in Figs. 7–10. The compression curves have been plotted to the ultimate buckling stress, the magnitude of which is dependent upon the dimensions of the test specimens. It is really the potential strength, and, as will be demonstrated later, it can probably be developed only if the structural member is designed against secondary or local buckling.

The tangent moduli determined for each stress-strain curve are given in Figs. 11, 12, 13, and 14. Likewise the secant moduli are given in Figs. 15, 16, 17, and 18. The secant modulus of elasticity is the ratio of maximum stress to maximum strain and is always greater than the corresponding tangent modulus above the proportional limit. The tangent and secant moduli converge with the corresponding initial moduli at the proportional limit and, consequently, are all the same below this stress. It is possible for the tangent modulus to become zero at the ultimate tensile stress. The lowest tangent modulus that can be developed in compression tests corresponds to the ultimate buckling stress. None of the observed values were less than 2,000,000 lbs. per sq.in.

Column Specimens

Corrugated sheets were used to test the theory because the results of such tests would have universal application. A pattern used in commercial roofing, which nominally has 1.25 in. pitch and a 0.375 in. depth, was selected because it was the nearest size to standard aircraft corrugations available. The rotary-drum type corrugating machine that was used is eminently suited for corrugating soft steels but less so for the materials described herein because of their large amount of "spring-back." The dimensions of the corrugated specimens are given in Table III. The average pitch ranged from 1.35 to 1.39 in. in corrugations made from both materials. Corrugated specimens of material B had average depths of 0.232 to 0.250 in. as-cold-rolled and 0.212 to 0.231 in. as-stressed-relieved. Corrugations of material A were consistently shallower than those of material B, ranging from 0.210 to 0.212 in. as-cold-rolled and from 0.201 to 0.206 in. as-stress-relieved.

The corrugations described herein are of the common arc-and-tangent type rather than aircraft type made up of connected circle arcs. Blodgett[19] worked out mathematically precise methods for determining the moments of inertia of arc-and-tangent corrugations in terms of the pitch-depth ratio and the angle of the flat or tangent with respect to the neutral axis. A modification of this method was used to compute the properties of the present corrugations. The radius of gyration was obtained as follows:

$$I = C_6 btd^2 \tag{18}$$

where

I = moment of inertia, in.^{4}
C_6 = Blodgett's factor, which depends on the pitch-depth ratio and tangent angle
b = width of corrugated sheet, in.
t = thickness, in.
d = depth of corrugation, in.

The cross-sectional area can be computed by means of the following formula:

$$A = \lambda_6 bt \tag{19}$$

where A = cross-sectional area, sq.in., and λ_6 = development factor for corrugating based on the pitch-depth ratio and the tangent angle. The least radius of gyration is then

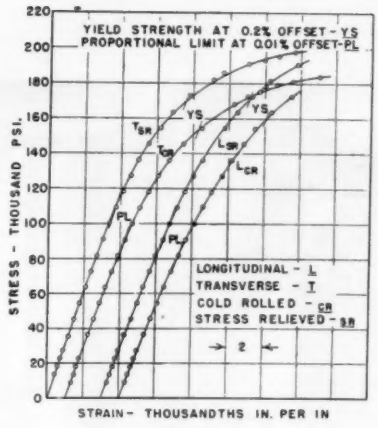
$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{C_6 btd^2}{\lambda_6 bt}} = d\sqrt{\frac{C_6}{\lambda_6}} \tag{20}$$

Four lengths of specimens for the tests were approximately as follows: (1) 20.5–21.0, (2) 10.2–10.9, (3) 4.9–5.2, and (4) 2.2–2.3 in. The ends of all specimens were cut square and parallel within about 0.01 in. by a stationary grinder fitted with a cutoff wheel.

Column Testing Procedure

Special apparatus was devised to give the corrugated specimens the minimum end restraint under the column loads. Fig. 19 shows one of the 20-in. long specimens being loaded with this apparatus in a 200,000-lb. Baldwin-Southwark universal testing machine. The top and bottom supports are sharp 90° hardened and ground knife edges that rest in shallow grooves, ground into the channel-shaped adapters. The knife edges extend the full length of 14 in. and withstood over 50,000 lbs. during the tests without noticeable wear. The top support was bolted to an adapter ring around a plane head resting in a spherical seat, affording a small measure of accommodation.

The knurled screws along the adapters are used to press the angles against the specimen in order (1) to hold assembly together while being placed in the ma-


YIELD STRENGTH AT 0.2% OFFSET - YS
PROPORTIONAL LIMIT AT 0.01% OFFSET - PL
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TRANSVERSE - T
COLD ROLLED - CR
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STRESS - THOUSAND PSI.
STRAIN - THOUSANDTHS IN. PER IN

Fig. 7 - STRESS-STRAIN CURVES FOR MATERIAL A IN TENSION

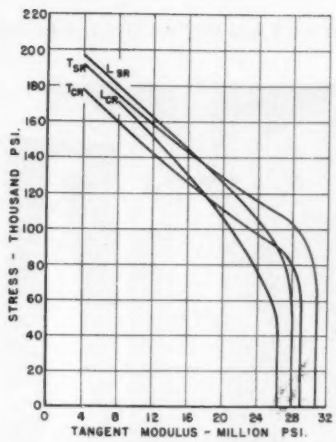


Fig. 11 - TANGENT MODULUS CURVES FOR MATERIAL A IN TENSION

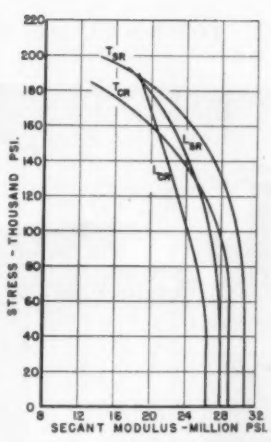


Fig. 15 - SECANT MODULUS CURVES FOR MATERIAL A IN TENSION

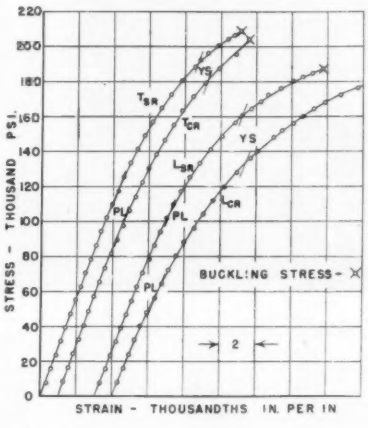


Fig. 8 - STRESS-STRAIN CURVES FOR MATERIAL A IN COMPRESSION

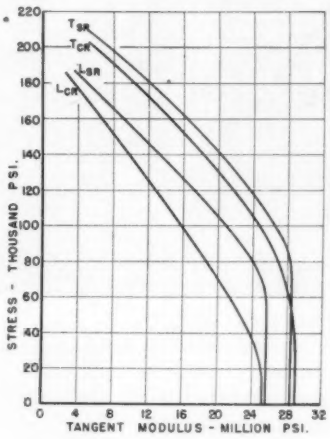


Fig. 12 - TANGENT MODULUS CURVES FOR MATERIAL A IN COMPRESSION

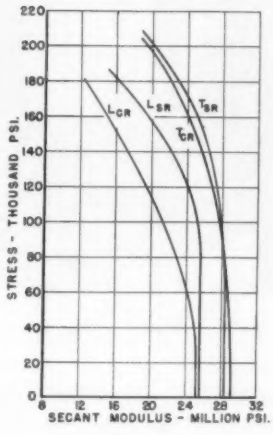


Fig. 16 - SECANT MODULUS CURVES FOR MATERIAL A IN COMPRESSION

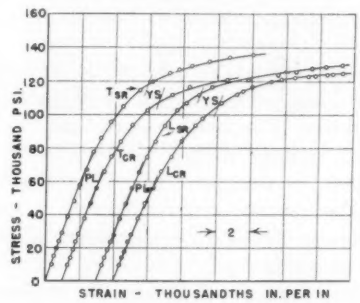


Fig. 9 - STRESS-STRAIN CURVES FOR MATERIAL B IN TENSION

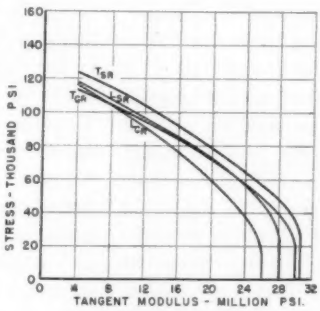


Fig. 13 - TANGENT MODULUS CURVES FOR MATERIAL B IN TENSION

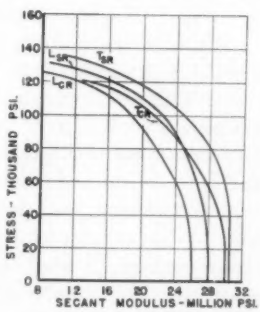


Fig. 17 - SECANT MODULUS CURVES FOR MATERIAL B IN TENSION

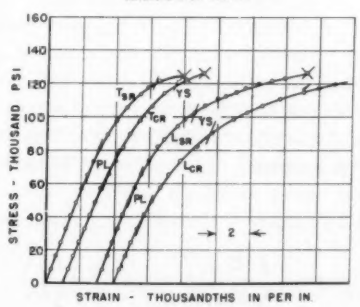


Fig. 10 - STRESS-STRAIN CURVES FOR MATERIAL B IN COMPRESSION

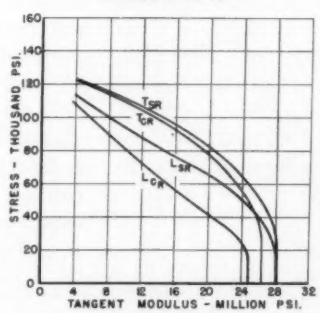


Fig. 14 - TANGENT MODULUS CURVES FOR MATERIAL B IN COMPRESSION

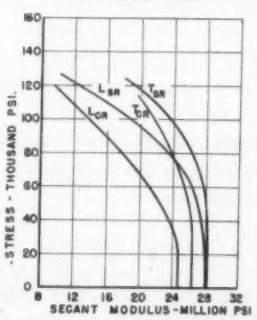
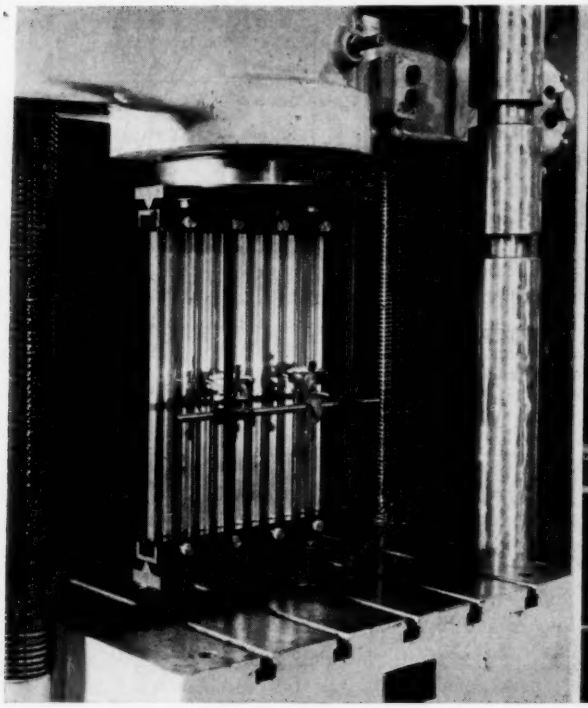


Fig. 18 - SECANT MODULUS CURVES FOR MATERIAL B IN COMPRESSION



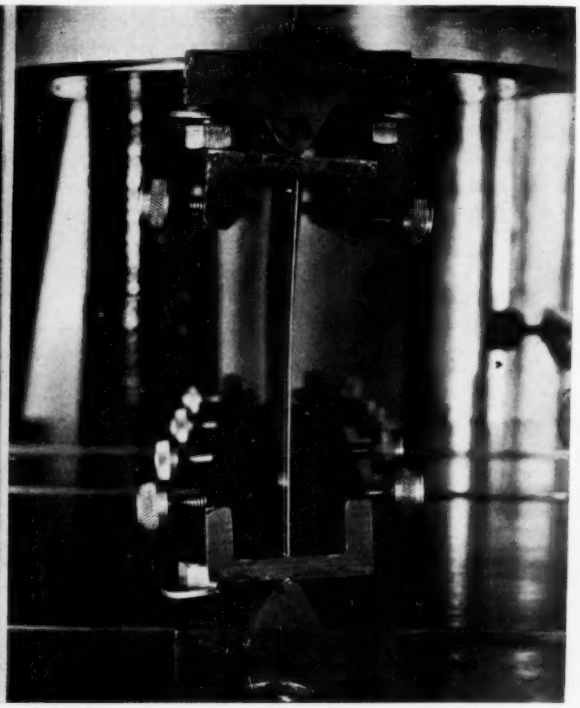


FIG. 19 (*left*). Column testing setup in 200,000-lb. Baldwin-Southwark testing machine. FIG. 20 (*right*). End view of testing setup with 5-in. specimen in place.

chine and (2) to adjust the position of the specimens with respect to the knife edges. The dial gauges were used to measure the lateral deflection of the specimen under load and adjust for minimum eccentricity. This operation consisted of setting the dials to zero at a seating load of about 200 lbs. and observing their movement when the load was increased to about half the final critical load. The screws were adjusted until the lateral deflections were less than 0.001 in. at half the critical load. This procedure was followed by slowly loading the column until the critical load was reached, the dials giving advanced warning by their greatly accelerated movement. At the critical load a slight finger pressure applied near mid-length was sufficient to move the specimen laterally, particularly in the longer specimens.

In order to remove any chance of restraint in the shorter specimens, the adjusting angles were removed after the eccentricity adjustment was made and the test was continued as shown in Fig. 20. This illustrates a 5-in. specimen after primary buckling has occurred.

In computing effective column length 0.6 in. was added to the specimen length to compensate for the adapter thickness.

COLUMN TEST RESULTS

All except the shortest of the specimens failed by primary Euler buckling. The shortest specimens had slenderness ratios from 33 to 41. Typical examples of the secondary or local buckling which occurred in some of them are shown in Fig. 21.

All of the data given in Table III were obtained under the first load. The critical column stresses were computed by dividing these loads by the cross-sectional areas of the specimens.

The critical column stresses were plotted against slenderness ratios in Figs. 22 and 23 for materials A and B, respectively. The curves shown are the result of substituting tangent moduli in the Euler formula at various stress levels against which the resulting computed slenderness ratios were plotted. The critical column stresses obtained on specimens of material B at various slenderness ratios fall close to the computed curves. Such agreement is usually rather easy to show in long columns, much less so in short columns.[20, 21, 24, 25] The agreement between test values and the Euler curves based on tangent moduli was also good for material A, except for the shortest specimens, which did not quite develop the expected stresses.

There is reason to believe that the shortest specimens of material A may have failed locally before the primary Euler failure occurred. According to Wilson and Newmark,[22] the stress at which wrinkling occurs can be computed by means of the following expression:

$$F = \left(\frac{E}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{1-\mu^2}}\right)\left(\frac{t}{R}\right) \qquad (21)$$

where μ = Poisson's ratio, approximately = 0.3, R =

mean radius of shell, in., and other terms as previously defined.

Suppose the wrinkling stress is set equal to the critical column stress, thus:

$$\frac{\pi^2 E}{(L/r)^2} = F = \left(\frac{E}{\sqrt{3}}\right)\left(\frac{1}{\sqrt{1-\mu^2}}\right)\left(\frac{t}{R}\right)$$

Then

$$L/r = \sqrt{\pi^2\sqrt{3}\ \sqrt{1-0.3^2}\,(R/t)}$$

$$= 4.03\sqrt{R/t} \qquad (22)$$

The smallest radius of curvature of the corrugated specimens was about 0.5 in. at the arc and approximately 3 in. at the section intended for a flat. Using Eq. (22) the minimum slenderness ratio without secondary failure for 0.020-in. thick material would be 20 for the 0.5-in. radius and 49 for the 3-in. radius. These calculations serve as a rough guide to show that secondary wrinkling probably adversely affected the critical column stresses developed in the shortest specimens. This is a principle of which the designer must never lose sight.

The high strengths that are indicated by compression tests on specimens ideally proportioned against wrinkling cannot be attained in structures unless they too are ideally proportioned to prevent secondary wrinkling.

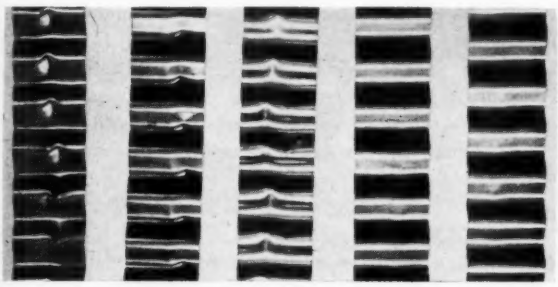


FIG. 21. Typical 2.25-in. columns showing secondary failure.

In general, the transverse specimens developed higher critical column stresses than the longitudinal ones. The benefits of stress-relieving appear to be slight. The variations in mechanical properties of anisotropic materials, such as high-strength stainless steel, have relatively little effect on behavior of short columns.

The stress analyst should familiarize himself with possible variations in tangent moduli of a given material in order to know the likely minimums that may be expected.

THEORY OF USE OF THE SECANT MODULUS

One of the uses of the secant modulus is in evaluating the collective action of structures made of two or more dissimilar materials fastened together as a unit, as in spar-sheet-stringer units of airplane wings. This leads to the solution of shear-lag problems.

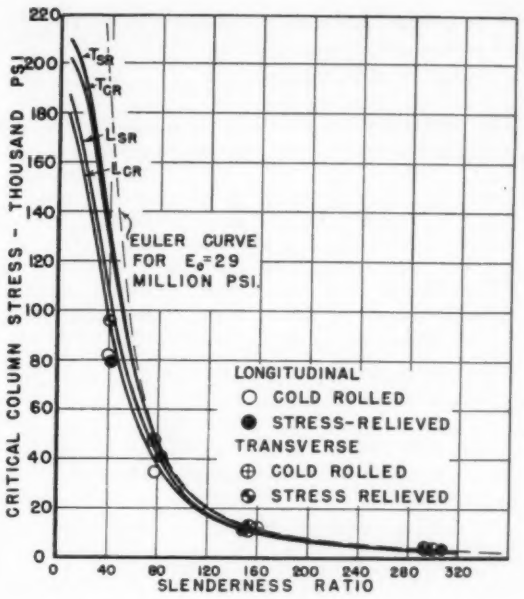


FIG. 22. Critical column stress versus slenderness ratio for specimens of material A.

Another use of the secant modulus is predicting deflections of hollow beams. According to Timoshenko,[23] the theory of bending beyond the proportional limit was first developed by Saint Venant and later augmented by Meyer and von Kármán. Timoshenko gives the equation relating the moment and the radius of curvature of a beam bent beyond the proportional limit as follows:

$$M = E_r I/\rho \qquad (23)$$

where

M = the moment, in.lbs.

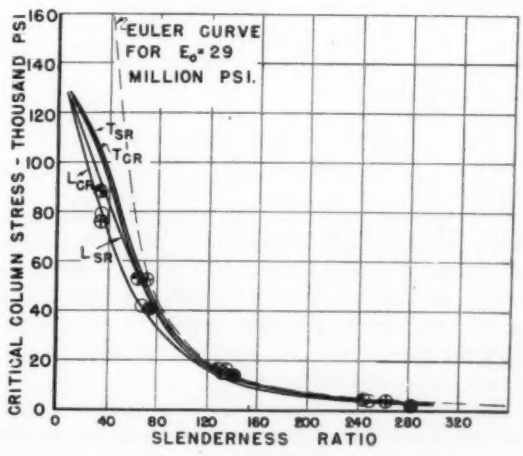


FIG. 23. Critical column stress versus slenderness ratio for specimens of material B.

E_r = reduced modulus of elasticity, lbs. per sq.in.
I = moment of inertia, in.^{4}
ρ = radius of curvature, in.

The expression for the reduced modulus of a solid rectangular section in bending can be given as follows:

$$E_r = (24/\Delta^3)\int_0^{\Delta/2} Fe\,de \tag{24}$$

where

Δ = difference in strain between top and bottom fibers, in. per in.
F = maximum stress, lbs. per sq.in.
e = maximum strain, in. per in.

In the above expression the integral is the area-moment of the stress-strain diagram about an axis through zero strain.

The determination of the area-moment of a stress-strain diagram can be handled by a modification of Simpson's rule, but the operation is tedious, making it somewhat impractical.

The derivation of the reduced modulus for a hollow section in which substantially all the material is remote from the neutral axis is as follows:

From the geometry of Fig. 2, it is seen that

$$dL/y = L/\rho \tag{25}$$

where dL is the change in length of the distance L at a distance y from the neutral axis. By definition

$$e = dL/L \tag{26}$$

By substitution

$$e = y/\rho \tag{27}$$

From Eq. (16), when $h = 2y$

$$I = tbh^2/2 = 2tby^2 \tag{28}$$

By definition, the secant modulus of elasticity is

$$E_s = F/e \tag{29}$$

The total moment is

$$M = 2F(tb)y = 2eE_s(tb)y = 2E_s(tb)y^2/\rho = IE_s/\rho \tag{30}$$

This equation shows that the reduced modulus of a hollow section with substantially all of its material remote from its neutral axis is equal to the secant modulus E_s. In practice, sufficient accuracy in determining the effective secant modulus should be obtained by averaging the values at a given stress level for both tension and compression.

The user of secant modulus data must remember that the reduced modulus can be applied directly only in the case of pure bending. Timoshenko deals with the method of handling other cases.[23]

Summary and Conclusions

(1) A double-modulus formula for determining strength of columns based on the initial and tangent moduli of elasticity has been developed for hollow sections in which substantially all of the material is remote from the neutral axis. This augments the formulas previously worked out by Considère, Engesser, and von Kármán for columns of solid rectangular cross section, as well as miscellaneous sections given by Osgood and others. These formulas enable the stress analyst to determine the probable critical loads in columns of materials worked above the proportional limit, using the classic Euler equation.

(2) Complete tension and compression tests were made on two tempers of 25 gauge Type 301 (17–7) high-tensile stainless steel. These tests were made on longitudinal and transverse specimens, both as-cold-rolled and after stress-relieving. Stress-strain, tangent, and secant modulus data are given in curve form.

(3) Column tests were made on corrugated sheets of the same stainless steel. These tests were made on longitudinal and transverse specimens, both as-cold-rolled and after stress-relieving. Specimens having slenderness ratios ranging from approximately 30 to 300 were loaded as columns with knife-edge end conditions.

(4) The critical column stresses were plotted against their corresponding slenderness ratios. Euler curves based on tangent moduli for the same materials were generally in good agreement with the experimental data. The failure of the shortest columns (having slenderness ratios around 35) to develop the stresses expected on the basis of tangent modulus data can be explained by premature crippling arising from local instability.

(5) The tangent modulus can be used directly in Euler's formula to design both solid and hollow columns. This results in sufficient accuracy without involving the use of the complicated reduced moduli.

(6) The inherent nature of short columns appears to minimize the importance of the somewhat wide variations in tangent moduli with respect to grain direction and stress-relieving. This works in favor of anisotropic materials such as cold-reduced austenitic and ferritic steels.

(7) The designer must not overlook the fact that the stresses indicated by mechanical tests on ideal specimens of high-strength materials can be developed only if local instability of the thin sections is taken into account.

(8) The reduced modulus to be used in determining the degree of bending in hollow sections with most of the material remote from the neutral axis is mathematically shown to be the secant modulus.

Acknowledgments

The data for this paper were determined by members of the Research Laboratories of The American Rolling

Mill Company and permission to publish has been given by this company.

The author is particularly indebted to Harry LaTour for his help in performing the tests and computing the data. Mr. Clark Prudhon's aid in developing rapid methods for measuring moduli was also inestimable. The consistency of the cylinder and column test data was, to a considerable extent, due to the methods and care exercised by J. C. Young and his staff in preparing the specimens. The valuable suggestions made by R. L. Kenyon regarding the presentation of the data should not go unmentioned.

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Elevated Temperature Aging of 24S Aluminum Alloy

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SUMMARY

The aging of strong aluminum alloys at elevated temperatures is not a new process and the attainable physical properties have been studied for a number of the alloys used in aircraft construction. However, the fear of corrosion difficulties and the lack of detailed information regarding the physical properties obtainable and the optimum conditions of time and temperature required to attain these desired properties for 24S alloy have delayed the application of this process in aircraft fabrication up to this time.

Various combinations of time and temperature will produce similar results, but the most satisfactory treatment for 24S aluminum alloy appears to be at 375°F. for 8 hours. This treatment will raise the tension yield point of the material approximately 50 per cent and the ultimate tensile strength about 10 per cent, while at the same time the elongation is reduced to approximately 5 per cent in 2 in. In general, after aging treatments, the physical properties in compression are slightly lower than in tension.

The corrosion resistance of aluminum-coated sheet is not seriously impaired but that of bare 24S alloy is appreciably lowered. Elevated temperature aging of bare 24S in thicknesses below $^3/_{16}$ in. is not advisable because of the increased tendency for intercrystalline corrosion.

The maximum physical properties obtainable by aging with any combination of time and temperature are largely dependent on the elastic strain present as a result of cold-work performed after the solution heat-treatment and before the elevated temperature aging. In order to assure consistent physical properties after an elevated temperature aging has been performed, it is necessary to control the degree of cold-working which the material undergoes during its manufacture.

INTRODUCTION AND HISTORY

THE FACT THAT AGING at elevated temperatures will appreciably change the physical properties of the duraluminum type alloys is not new. Numerous reports on the results obtainable by aging treatments on 17S alloy used in this country and various similar alloys used in foreign countries, including the so-called "Superdural," have been available for many years in the published literature. Some of the earlier investigations[1] covered the physical properties obtainable by elevated temperature aging and studied the effects of the use of different temperatures for different periods of time, in addition to the corrosion resistance characteristics of the alloys which resulted from the various treatments investigated. Duraluminum type alloys containing appreciable amounts of silicon have been reported to exhibit poor corrosion resistance if aged between approximately 250° to 280°F. Later studies[2] determined the relationships between time and temperature for attaining maximum hardness and the several different phases of precipitation resulting from elevated temperature aging were reported. The investigations referred to above were made on material that had been given a solution heat-treatment before aging, but no residual stresses were present in the material other than those caused by quenching during the heat-treatment. Numerous later reports included a study not only of duraluminum type alloys that had been aged without mechanical working but also included a study of the effects of aging after cold-work. Since it is also known that the different duraluminum type alloys do not reach the same physical properties after similar aging treatments and since no complete study of 24S alloy had previously been made, the investigation covered in this paper was undertaken to establish definite physical properties that could be expected and to determine the optimum combination of time, temperature, and amount of cold-work necessary to produce those properties. The necessity for higher physical properties in the aluminum alloys used for structural purposes has been recognized for many years, and it is believed that the results indicated in this study may be applied to aircraft fabrication without the necessity for extensive changes in processing equipment and without any changes in present tooling methods.

Since the usual mill practice is to introduce a certain amount of cold-working into 24S sheet material and extruded sections during the straightening procedure following the solution heat-treatment, particular attention was given at first to the properties that could be obtained by elevated temperature aging of the usual mill-run product. Such material, however, as reflected in the degree of residual elastic deformation present, proved to be too inconsistent, and that portion of the investigation was discontinued. The remainder of the investigation was confined almost entirely to the study of the time and temperature necessary to develop the desired properties in material that had been stretched in the laboratory to 1 per cent permanent set after the solution heat-treatment in order to determine what could be obtained by more closely controlled conditions. This amount of cold-work is approximately the same that would normally be expected during mill production of the material, and

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for this reason the data that have been obtained represent the results that may be expected with consistently controlled mill practice.

Type of Precipitation and Controlling Factors

Effect of Cold-Work

Contrary to general opinion that intercrystalline corrosion susceptibility will result in 24S alloy if heated for any length of time above 212°F., no serious corrosion difficulties were observed in the aluminum-coated material when aged at a much higher temperature. This was especially true of specimens that had been strained to 1 per cent permanent set after solution heat-treatment but before aging in which the precipitation was of a general nature along the highly stressed slip planes and was not observable to any appreciable extent in the grain boundaries. Bare 24S material, however, was susceptible to intercrystalline attack if aged for a longer time than necessary to reach the peak physical properties. This indicates the presence of grain boundary precipitation. These results are in agreement with the observations reported by Brenner[3] several years ago. Data are not available on the percentage of reduction by rolling necessary to produce the elastic strain obtained by 1 per cent permanent set produced by stretching, but a few scattered values that have been obtained from other investigations would indicate that approximately 2 per cent reduction by rolling would be necessary to produce the conditions for general or slip-plane precipitation required to attain the physical properties obtained in this investigation by stretching to 1 per cent set. Fig. 1 illustrates the effect on the physical properties of different degrees of stretching before the aging treatment.

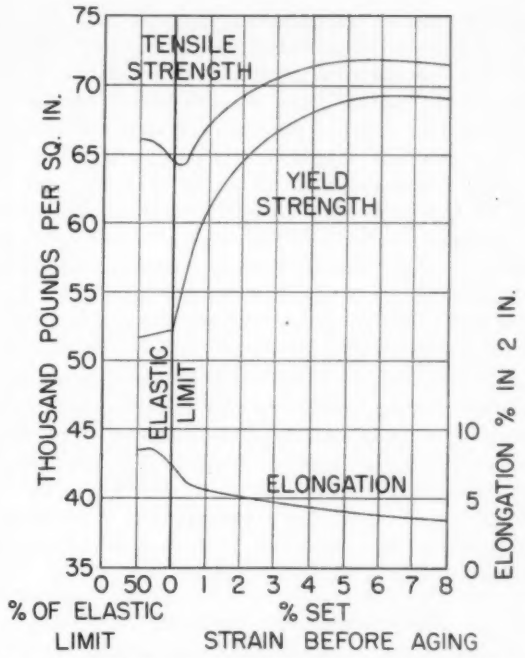

FIG. 1. Physical properties obtained in 24S-O Alclad sheet, solution heat-treated at 925°F., then subjected to different amounts of strain by stretching and aged for 20 hours at 350°F.

Effects of Different Temperatures

No appreciable change in strength characteristics or corrosion resistance resulted from aging at temperatures below 300°F. for 20 hours, and apparently any effects to be produced within a length of time that would be practicable for shop operations must be obtained by aging at temperatures well above 300°F. An exploration of the temperature range that had been found to cause undue loss in corrosion resistance in certain alloys was conducted at temperatures from 150° to 350°F. at 25° intervals. No critical temperature zone could be found which would produce any unusual degree of susceptibility to corrosion.

The speed of precipitation depends upon molecular motion which, of course, depends on the temperatures used. Apparently, an increase in the aging temperature above 350°F. slightly lowers the tensile properties that can be obtained, but at the same time the maximum physical properties will be reached in a shorter length of time. As will be seen in Fig. 2, the time to reach the maximum tensile yield point is approximately

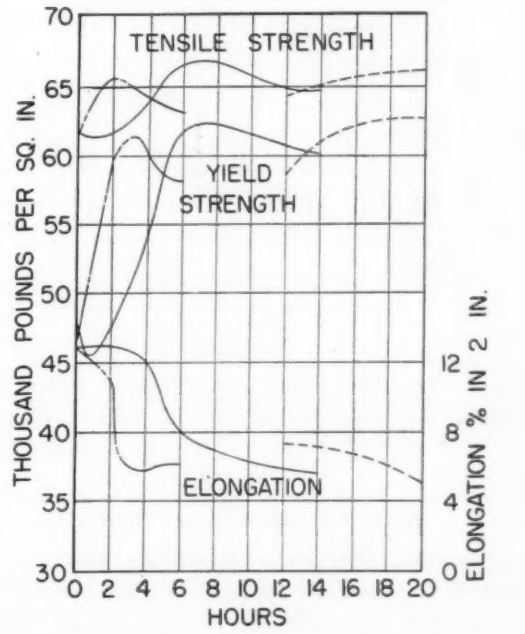

FIG. 2. Physical properties obtained in 24S-O Alclad, solution heat-treated at 925°F., stretched to 1 per cent permanent set, and aged at 350°, 375°, or 400°F. for various periods of time. (Broken line, 400°F.; solid line, 375°F.; dotted line, 350°F.)

cut in half with each 25°F. rise in temperature above 350°F. At first thought, this would appear to indicate that temperatures high enough to shorten the aging time to a low period could be permitted; how-

ever, the use of temperatures above 375°F. cause a peak in physical properties to be reached very rapidly and then the physical properties decrease quickly to much lower values. Any such treatment would be considered too critical for practical shop application. At the same time excessive precipitation and lowered corrosion resistance was noted in samples aged at 400°F., indicating that aging temperatures should be kept somewhat lower. The effects of overaging as indicated above are to decrease both the corrosion resistance and the physical properties. From this same Fig. 2, it may be seen that the selection of 375°F. for aging operations in the shop combines both a reasonable time limit (7 to 8 hours) and a wide enough range in temperature to permit ordinary variations in furnace operation without damaging the material. The time necessary to attain maximum tensile yield strength at any aging temperature is shown by Fig. 3, and by use of this figure the required combination of time and temperature for obtaining the maximum desired properties may be easily determined.

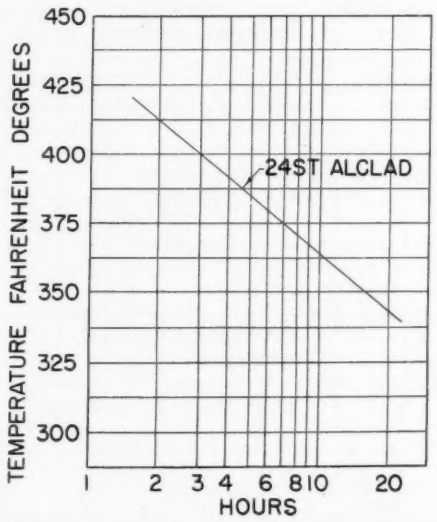

FIG. 3. The relation between time and temperature required to obtain maximum tensile yield strength (0.2 per cent set), in 24S Alclad sheet, solution heat-treated at 925°F. and stretched to 1 per cent permanent set before aging.

PHYSICAL PROPERTIES ATTAINABLE

As has been previously mentioned, the physical properties that may be obtained by aging at temperatures above 300°F. depend to a large extent on the amount of cold-working to which the material has been subjected after solution heat-treatment and before the elevated temperature aging. Table 1 illustrates the values found in specimens that had been solution heat-treated and aged without subsequent cold-work compared to the results that were obtained in specimens cut from the same sheet of raw stock and aged after having been given 1 per cent permanent set by stretching. It will be noted that it is possible to obtain a gain in tensile strength of approximately 7 per cent and again in tensile yield strength of approximately 30 per cent when the unstretched specimens were aged at 350°F. for 20 hours. Approximately the same values would be expected after aging at 375°F. for 8 hours. In contrast, material that had been stretched 1 per cent before aging showed a gain of almost 10 per cent in tensile strength and 55 per cent gain in tensile yield strength upon aging for 20 hours at 350°F. While little additional percentage gain in ultimate tensile strength is obtainable by strain hardening the alloy before aging, almost double the gain in yield strength is obtainable when only 1 per cent permanent set has been given the material. Much higher values will be noted for 24S-RT material that had been cold-rolled after heat-treatment by the mill. The percentage of cold-working that such material had received is not known, but properties attained by aging were comparable to those found in specimens that had been stretched about $5^1/_2$ per cent before aging. Thus it will be seen that in order definitely to establish design values to be used for artificially aged 24S material it will be necessary to establish a minimum amount of cold-working to be performed on the material after the solution heat-treatment. This, of course, may either be performed at the mill or, in some instances, in the fabrication shop. In the case of large sheets, control must be exercised at the mill.

TABLE 1

Maximum Properties Found

Stretch Before Aging, %	Aging Treatment	Tensile Strength, Lbs. per Sq.In.	Increase in Tensile Strength, %	Yield Strength, Lbs. per Sq.In.	Increase in Yield Strength, %
0	0	61,460	0	39,630	0
0	12 hr. 350°F.	63,570	3.5	47,070	18.8
0	20 hr. 350°F.	65,830	7.2	51,470	30.0
1	20 hr. 350°F.	67,570	9.9	61,180	54.5
1	7.75 hr. 375°F.	67,740	10.2	62,570	57.9
1	3.5 hr. 400°F.	65,000	5.8	61,460	55.1
S-RT*	0	67,890	0	54,410	0
S-RT*	12 hr. 350°F.	74,460	9.7	71,260	31.0
S-RT*	5 hr. 375°F.	73,630	8.5	71,160	30.8
S-RT*	2 hr. 400°F.	72,570	6.9	69,610	28.0

* The per cent cold-working of the 24S-RT aluminum-coated sheet used is not known.

Elongation

Since the elongation in 2 in. of all specimens that were aged to a maximum yield strength regardless of their condition of cold-work, temperature, or time involved reached a value of approximately 5 per cent in 2 in., it is believed that this value must be accepted as a minimum value for the elongation of material that has been properly aged. Any attempt to stop the aging at some point that would produce increased elongation would mean an attempt to control the time on a portion of the curve where the yield strength is rising rapidly and would certainly result in uncontrollable values for all physical properties other than

the elongation itself. It is realized that this low elongation in some cases may be undesirable from a design standpoint but it is felt that this value must be accepted and any use of material that has been aged at elevated temperatures must take this fact into consideration at the time the design is made. Since the shear strength follows the changes in ultimate tensile strength rather than yield strength, only slight increases have been found; in fact, not enough to be of any great interest from a design standpoint.

Compressive Yield Strength

The compressive yield strength, which is perhaps the most important property for consideration in the use of 24S sheet, has not been thoroughly studied, but the few tests that have been made indicate that for material that has been given 1 per cent permanent set the values are only slightly below the tensile yield strength and in material that has been stretched to produce results comparable to 24S-RT the compressive yield strength is slightly higher than the tensile yield strength.

Fatigue Tests

Fatigue tests have been made on specimens cut from extruded sections, and little change in fatigue strength has been found in the aged versus unaged material. This is in agreement with data that have been published for 14S forging alloy, which is a similar type of material.[4]

No data are available on notch sensitivity or on impact properties at the present time. There is no reason, however, to believe that the behavior of 24S alloy properly age-hardened would exhibit different characteristics from those reported for 14S, which indicate that the aged specimens fail at approximately 2,000 lbs. per sq.in. below the values for unaged material.

Tests conducted on samples cut from an extrusion having both heavy and thin sections indicate that somewhat better properties may be obtained in the heavier portion of the section. These tests, however, were all made at 350°F. and may not be true if other aging temperatures are used. These results are given in Table 2.

TABLE 2

Maximum Properties Obtained by Aging Treatments on 24S-T Extruded Sections. Thick and Thin Sections Taken from Leg and Body of Same Extrusion

Aging Treatment	Tensile Strength, Lbs. per Sq.In.	Tensile Strength, Lbs. per Sq.In.	Compression Yield Strength, Lbs. per Sq.In.	Type of Specimen
20 hr. 350°F.	68,300	62,950	....	0.133-in. strip
20 hr. 350°F.	....	....	60,030	0.133-in. strip
16 hr. 350°F.	....	....	66,600	0.798 in. in diameter, 2⅜ in. length

Corrosion Resistance

Stress Corrosion

Some apprehension was felt regarding the possible susceptibility for stress corrosion that might be introduced by elevated temperature aging and, in order to study this question, a number of samples were prepared by spot welding and by riveting. A portion of the specimens were assembled before aging and others were aged before assembly. In order to expose the bare material, the protective aluminum layer was removed by countersinking on the heavier gauges, while in the lighter gauges the sheets were dimpled before riveting. It was believed that if any serious susceptibility to stress corrosion was present it would be indicated by rapid joint failure of the assembled specimens. These samples were then placed in a stressed condition by bending into a bow and were immersed in a salt-peroxide solution as the corroding medium. Tests indicate that no susceptibility to stress corrosion is caused by artificial aging. Because of the fact that aluminum-coated material in the thinner gauges will not have complete anodic protection by the pure aluminum coating, because of the diffusion zone extending either entirely or almost to the surface, it is not considered advisable to artificially age aluminum-coated material in thicknesses below 0.032 gauge.

General Corrosion

Duplicate specimens were prepared representing the different aging treatments in order that one set could be tested to determine the loss in physical properties due to corrosion susceptibility caused by the different aging treatments. The loss of properties resulting from accelerated corrosion tests in a salt-peroxide solution for 115 hours is shown in Fig. 4 and Fig. 5. From these curves it will be seen that nothing is to be feared in the case of aluminum-coated material and that no serious difficulties would be expected in heavy gauges of bare 24S material, unless it had been overaged. The relatively large notch effect of corroded areas on small test specimens as compared to actual aircraft parts is reflected in the loss in elongation and tensile strength as shown in Figs. 4 and 5.

Design Possibilities

Because of the fact that any specification prepared for the purchase of material intended for elevated temperature aging would require tests to be performed at the mill laboratory under closely controlled conditions, it is believed advisable to keep the values used for design purposes well below those specified for procurement purposes. This will permit reasonable deviations in shop practice, normal variations in furnace temperatures, and a reasonable variation in aging time and still assure the manufacturer that his material processed under shop conditions will have

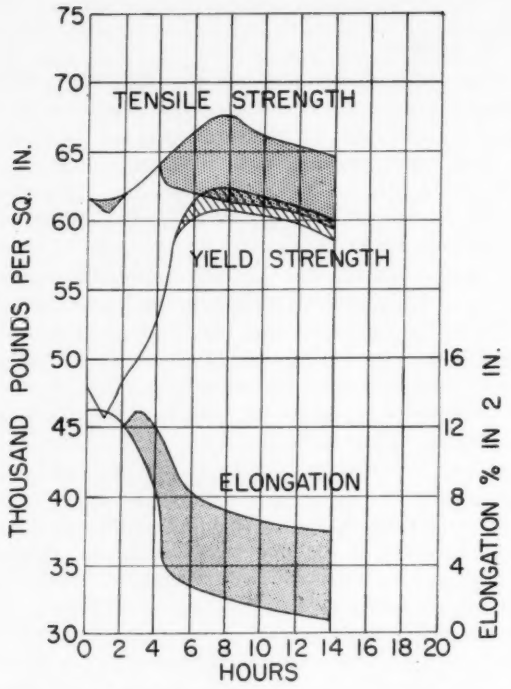

FIG. 4. The effect on corrosion resistance of 24S-T Alclad resulting from aging at 375°F. for various periods of time to 14 hours as indicated by physical properties after alternate immersion in salt-peroxide solution for 115 hours compared to the physical properties of noncorroded specimens. Cross hatching indicates loss in properties.

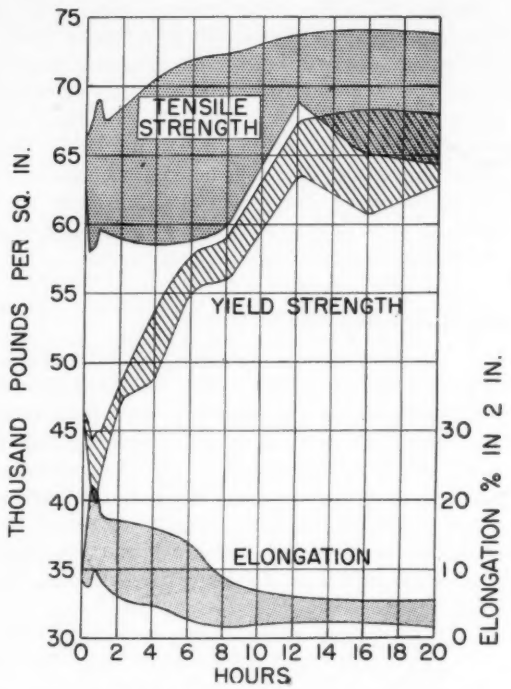

FIG. 5. The effect of aging at 350°F. for various periods of time to 20 hours on the corrosion resistance of bare 24S-T sheet as indicated by the physical properties after 115 hours alternate immersion in salt-peroxide solution compared to the physical properties of noncorroded specimens. Cross hatching represents loss in properties.

properties well above the values used for design purposes.

CONCLUSIONS

The tensile yield strength, ultimate tensile strength, and compressive yield strength of 24S aluminum-coated sheet may be increased by aging at elevated temperatures without appreciably lowering the corrosion resistance of the material in the case of aluminum-coated sheet. These values are further increased by a slight amount of plastic deformation before the aging treatment. The increase in properties depends upon the degree of plastic deformation, the aging temperature, and the time at the aging temperature. The optimum properties may be obtained on material that has been stretched to a permanent set of 1 per cent in approximately 16 hours at 350°F. and in approximately 7 hours at 375°F. No effects of elevated temperature aging other than the decrease in elongation should cause any apprehension in the use of elevated temperature aging in the fabrication of aircraft parts from aluminum-coated 24S sheet in gauges of 0.032 in. or heavier or the bare alloy in relatively heavy sections.

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Aerodynamic Performance of the Towed Glider

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SUMMARY

The towed glider or the aerial train as it is sometimes called has been employed successfully for war purposes under special conditions. Such successful military use and the increase in pay load which follows when a powered airplane is used to tow a string of gliders have led to optimistic evaluation of the glider train both for military and commercial purposes. The purpose of this paper is to investigate the performance of the towed glider to see whether these expectations are likely to be realized.

In Part I of this paper certain aerodynamic theorems have been set down which serve as a useful introduction to the subject. Part I concludes by the discussion of some indications regarding towed glider design and performance which are apparent from the theorems themselves.

In Part II calculations have been carried out of the performance of a conventional transport airplane to which one, two, three, or four gliders are attached.

Part III is a discussion of the calculated results, with some consideration of how the powered airplane might be modified to make it more useful in a glider train.

SYMBOLS USED

	Towing Airplane or Tug	Glider or Tow
Gross weight, lbs.	W_a	W_g
Ratio of gross weights	$W_g/W_a = r_w$	
Number of gliders		n
Fuel and oil loads, lbs.	Wf_a	Wf_g
Weight of crew, lbs.	W_{ca}	W_{cg}
Pay load, lbs.	W_{pa}	W_{pg}
Disposable load, lbs.	$Wd_a = Wf_a + W_{ca} + W_{pa}$	$Wd_g = Wf_g + W_{cg} + W_{pg}$
Empty weight, lbs.	$We_a = W_a - Wd_a$	$We_g = W_g - We_f$
Wing area, sq.ft.	S_a	S_g
Effective aspect ratio	R_{ea}	R_{eg}
Span	b_a	b_g
Effective span, ft.	$e_a b_a$	$e_g b_g$
Ratio of wing areas	$S_g/S_a = r_s$	
Brake horsepower of engine	b.hp.	
Thrust horsepower	t.hp. = b.hp.η	
Propulsive efficiency	η	
Air density	ρ	
Lift coefficient	C_{L_a}	C_{L_g}
Combined lift coefficient	C_{L_t}	
Profile + parasite drag coefficient	$C_{D_{pa}}$	$C_{D_{pg}}$
Induced drag coefficient	$\frac{C_{L_a}^2}{R_{ea}} = C_{D_{ia}}$	$\frac{C_{L_g}^2}{R_{eg}} = C_{D_{ig}}$
Profile drag coefficient	$C_{D_{0a}}$	$C_{D_{0g}}$
Parasite drag coefficient	$C_{D_{para}}$	$C_{D_{parg}}$
Total drag coefficient	C_{D_a}	C_{D_g}
Combined drag coefficient	C_{D_t}	
Lift/drag	$(L/D)_a = E_a$	$(L/D)_g = E_g$
Thrust, lbs.	T_a	T_g
Combined thrust, lbs.	T_t	
Wing loading, lbs. per sq. ft.	$W_a/S_a = w_a$	$W_g/S_g = w_g$
Ratio of wing loading	$w_g/w_a = r_w$	
Air speed, ft. per sec.	V	
Actual climb speed in ft per sec.	V_c	
Dynamic pressure	$q = \rho V^2/2$	
Fictitious sinking speed	V_{s_a}	V_{s_g}
Combined fictitious sinking speed	V_{s_t}	
Fictitious climbing speed	V_{fc_a}	V_{fc_g}

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PART I—SOME THEOREMS IN THE PERFORMANCE OF THE TOWED GLIDER

Performance Equation of a Tug Without Gliders

THE TOTAL DRAG of the tug can be represented by the equation

$$D_a = D_{ia} + D_{0a} + D_{para} = \frac{\rho}{2}\left[(C_L{}^2{}_a/\pi R_{ea}) + C_{D_{pa}}\right] S_a V^2 \qquad \text{(I-1)}$$

In level flight, the velocity can be expressed by

$$V = [W_a/(\rho/2)C_{L_a}S_a]^{1/2}$$

Then the performance equation becomes

$$\text{t.hp.}_{\text{reqd.}a} = \text{b.hp.}_{\text{reqd.}a}\,\eta = \frac{\rho S_a}{1{,}100}\left[\left(\frac{C_{L_a}{}^2}{\pi R_{ea}}\right) + C_{D_{pa}}\right]\left[\frac{w_a}{(\rho/2)C_{L_a}}\right]^{3/2} \qquad \text{(I-2)}$$

Lift Coefficient of the Glider in Level Tow in Relation to the Lift Coefficient of the Airplane

$$V = [w_a/(\rho/2)C_{L_a}]^{1/2} = [w_g/(\rho/2)C_{L_g}]^{1/2}$$

so that

$$w_a/C_{L_a} = w_g/C_{L_g}$$

and

$$C_{L_g} = C_{L_a}(w_g/w_a) = C_{L_a}r_w \qquad \text{(I-3)}$$

Combined Drag Coefficient of a Glider Train in Level Tow in Terms of Tug Lift Coefficient

If, in the glider train, all gliders are alike, as is generally the case, the expression for the total drag becomes:

$$D_t = C_{D_t}(S_a + nS_g)q$$
$$= (C_{D_{i_a}} + C_{D_{p_a}})S_a q + n(C_{D_{i_g}} + C_{D_{p_g}})S_g q \quad \text{(I-4)}$$

By simple algebraic methods it can be shown that the expression for the combined drag coefficient is

$$C_{D_t} = \frac{C_{D_{p_a}} + nC_{D_{p_g}}r_s + \frac{{C_{L_a}}^2}{\pi}\left(\frac{1}{R_{e_a}} + \frac{nr_s{r_w}^2}{R_{e_g}}\right)}{1 + nr_s} \quad \text{(I-5)}$$

An alternative expression for C_{D_t} is:

$$C_{D_t} = (C_{D_a} + nC_{D_g}r_s)/(1 + nr_s) \quad \text{(I-6)}$$

Combined Lift Coefficient of Glider Train in Terms of the Tug Lift Coefficient

Since the total lift equals the sum of the lifts,

$$L_t = C_{L_a}S_a q + C_{L_g}nS_g q$$

so that,

$$C_{L_t} = (C_{L_a}S_a + C_{L_g}nS_g)/(S_a + nS_g) \quad \text{(I-7)}$$

and by appropriate substitution

$$C_{L_t} = C_{L_a}[(1 + nr_w r_s)/(1 + nr_s)] \quad \text{(I-8)}$$

Combined L/D in Terms of the Tug Lift Coefficient

$$(L/D)_t = \frac{C_{L_t}q(S_a + nS_g)}{C_{D_t}q(S_a + nS_g)} = \frac{C_{L_t}}{C_{D_t}} \quad \text{(I-9)}$$

and by appropriate substitution

$$(L/D)_t = \frac{C_{L_a}(1 + nr_w r_s)}{C_{D_{p_a}} + nC_{D_{p_g}}r_s + {C_{L_a}}^2\left(\frac{1}{R_{e_a}} + \frac{r_s{r_w}^2}{R_{e_g}}\right)} \quad \text{(I-9)}$$

It can be readily shown that the best L/D of a glider train occurs when

$$C_{D_{p_a}} + nC_{D_{p_g}}r_s = \frac{{C_{L_a}}^2}{\pi}\left[\frac{1}{R_{e_a}} + \frac{nr_s}{R_{e_g}}(r_w)^2\right] \quad \text{(I-10)}$$

Thus it is seen that, just as in the case of a single airfoil, the profile plus the parasite drag of the combination is equal to the induced drag of the combination when D/L is a minimum or L/D is a maximum.

It also can be shown that the best power coefficient of a glider train will occur under the same circumstances as the best power coefficient of a single airfoil or airplane.

Performance Equation of a Tug with Gliders

$$D_t = (\rho/2)C_{D_a}S_a V^2 + (\rho/2)C_{D_g}nS_g V^2$$
$$= (\rho/2)(C_{D_a}S_a + nC_{D_g}S_g)(2w_a/\rho C_{L_a})$$

Then

$$\text{t.hp.}_{\text{reqd.}_t} = \text{b.hp.}_{\text{reqd.}_t}\,\eta$$
$$= \left(\frac{\rho}{1{,}100}\right)S_a\left\{C_{D_{p_a}} + \frac{{C_{L_a}}^2}{R_{e_a}} + nr_s\left[C_{D_{p_g}} + \left(\frac{{C_{L_a}}^2}{\pi R_{e_g}}\right){r_w}^2\right]\right\}\left(\frac{2w_a}{\rho C_{L_a}}\right)^{3/2} \quad \text{(I-11)}$$

Comparing the power required for the tug alone Eq. (I-2) with that for a glider train Eq. (I-11), it can be seen that if the C_L of the airplane and the velocity are held constant the power required is increased when gliders are added. Also, if it is desired to hold the horsepower required constant, the velocity will have to be decreased and the C_L increased.

Expression for Total Thrust in Terms of the Lift/Drag Ratio

The combined thrust

$$T_t = T_a + nT_g = \frac{W_a}{(L/D)_a} + \frac{nW_g}{(L/D)_g} = \frac{w_a}{E_a} + \frac{nW_g}{E_g} \quad \text{(I-12)}$$

Expression for Total Power in Terms of the Lift/Drag Ratio in Level Flight

In level flight

$$\text{b.hp.}\eta(550) = (W_a V/E_a) + (nW_g V/E_g) \quad \text{(I-13)}$$

Fictitious Rate of Climb

Dividing both sides of Eq. (I-19) by W_a,

$$\frac{550\text{b.hp.}\eta}{W_a} = \frac{V}{E_a} + \left(\frac{nW_g}{W_a}\right)\left(\frac{V}{E_g}\right) = V_{fc_a} \quad \text{(I-14)}$$

where V_{fc} is the so-called fictitious rate of climb of the towing airplane. V_{fc} is the climb that would be obtained if the entire effective horsepower were utilized to overcome the force of gravity acting on the airplane.

Fictitious Sinking Speeds

It is convenient to introduce the term, fictitious sinking speed,

$$\left.\begin{aligned} V_{fs_a} &= V/E_a \\ V_{fs_g} &= V/E_g \end{aligned}\right\} \quad \text{(I-15)}$$

If V is the speed along the flight path, it can be seen that the sinking speed would be approximately

$$V/(L/D) = V/E$$

The fictitious sinking speed of the glider train as a whole is written as:

$$V_{fs_t} = V_{fs_a} + (nW_g/W_a)V_{fs_g} \quad \text{(I-16)}$$

Actual Rate of Climb of the Glider Train

The excess horsepower,

$$\text{ex.hp.} = 550\text{b.hp.}\eta - [(W_a V/E_a) + (nW_g V/E_g)]$$

Then, the actual rate of climb,

$$V_c = \left\{\frac{550\text{b.hp.}\eta}{W_a} - \left(\frac{V}{E_a} + \frac{nW_g V}{W_a E_g}\right)\right\}\frac{W_a}{W_a + nW_g}$$

$$= \left(V_{fc_a} - V_{fs_t}\right)\frac{W_a}{W_a + nW_g} \qquad \text{(I-17)}$$

As can be seen from Eq. (I-17), the maximum rate of climb will occur when the difference between the fictitious climbing speed and fictitious sinking speed is a maximum.

Equation for Level Flight in Terms of Sinking Speeds

$$V_{fc_a} - V_{fs_t} = \frac{550\text{b.hp.}\eta}{W_a} - \left[V_{fs_a} + \left(\frac{nW_g}{W_a}\right)V_{fs_g}\right] = 0 \qquad \text{(I-18)}$$

since the actual rate of climb is then zero.

Graphic Representation of Sinking and Climbing Speeds

A graphic representation of the ideas brought forth in the preceding paragraph is given in Fig. 1 in a speed polar diagram. In this diagram, V_{fs_g} is plotted against V in the upper curve and V_{fs_a} in the lower curve. Also in the lower portion $(nW_g/W_a)V_{fs_g}$ is added to V_{fs_a} to given V_{fs_t}. If V_{fc} is likewise plotted against V, the intersection determines the level flight condition. When V_{fc} falls below V_{fs_t}, there is climb; when V_{fc_a} falls above V_{fs_t}, there is descent.

Factors Entering into Decrease of Speed Caused by Towed Gliders

The speed polar diagram shown in Fig. 1 readily indicates the factors that produce decrease in speed and climb when the tow is connected to the tug. They are: (1) V_{fs_g}, the sinking speed of a single glider—in other words, its aerodynamic efficiency; (2) the product nW_g/W_a, which is the ratio of the total weight of the gliders to the weight of the tug.

Characteristic Points on the Speed Polar Diagram

In Fig. 2 the combined fictitious sinking speed curve, V_{sf_t}, and V_{fc_a} are drawn. The intersection of the two curves determines the maximum speed. The maximum difference between V_{sf_t} and V_{fc_a} fixes the maximum rate of climb and gives the speed for best climb.

Since the distance covered is proportional to V and since the power and, hence, the fuel consumption is proportional to V_{fs_t} in level flight, the maximum range will be obtained at the speed when V/V_{fs_t} is a maximum. Therefore, the speed for maximum range is found by drawing a line from the origin tangent to the V_{sf_t} curve. In a tail wind, $+V_w$, the origin is shifted to the left, and the opposite applies to a head wind, $-V_w$.

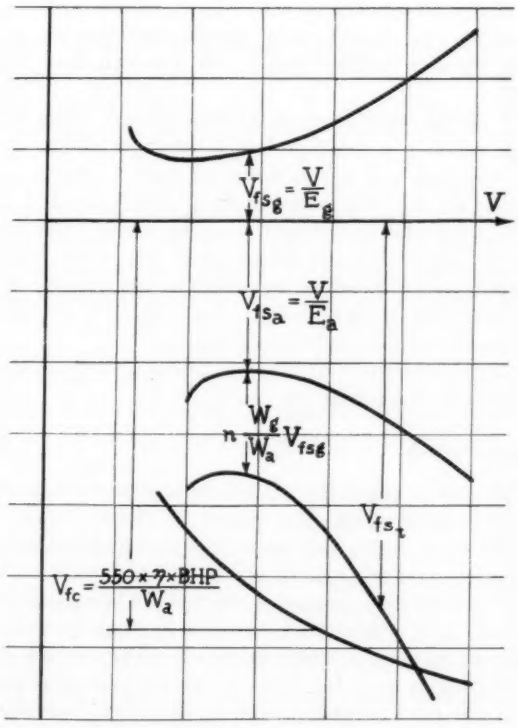

FIG. 1.

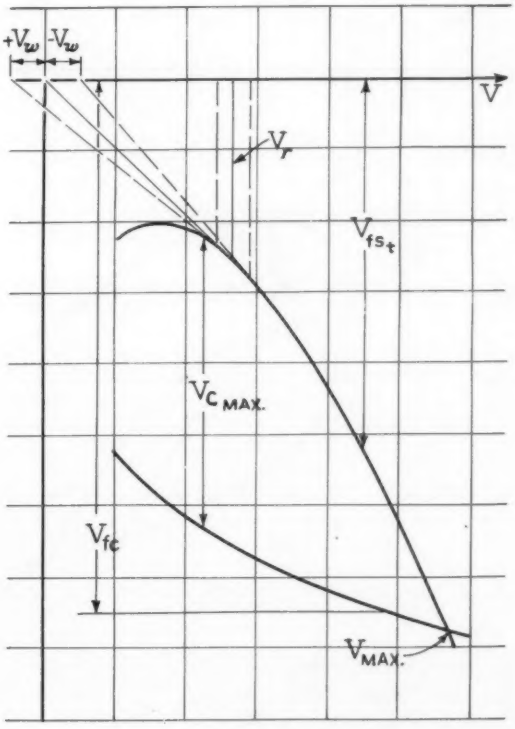

FIG. 2.

Here it is tacitly assumed that the specific fuel consumption is independent of the variation in the velocity V.

Effect of Change of Glider Wing Loading on Performance Speed Polar Diagram

The fictitious sinking speed, V_{fs}, may be written

$$V_{fs} = \frac{V}{E} = \left(\frac{2w}{\rho C_L}\right)^{1/2}\left(\frac{C_D}{C_L}\right) = \left(\frac{2w}{\rho}\right)^{1/2}\frac{C_D}{C_L^{3/2}} \qquad \text{(I-19)}$$

while

$$V = (2w/\rho)^{1/2}(1/C_L^{1/2})$$

(It will be recalled that $C_D/C_L^{3/2}$ is the power coefficient of conventional performance theory.)

At the same attitude and value of C_L, both the speed along the flight path and the fictitious sinking speed, at a constant gross weight, will be proportional to the square root of the wind loading ratio. This effect is shown in Fig. 3, where an increase in the wing loading of the tow from w' to w'' merely shifts the speed polar diagram to the right and upward—that is, if no change in external form accompanies the change in wing loading.

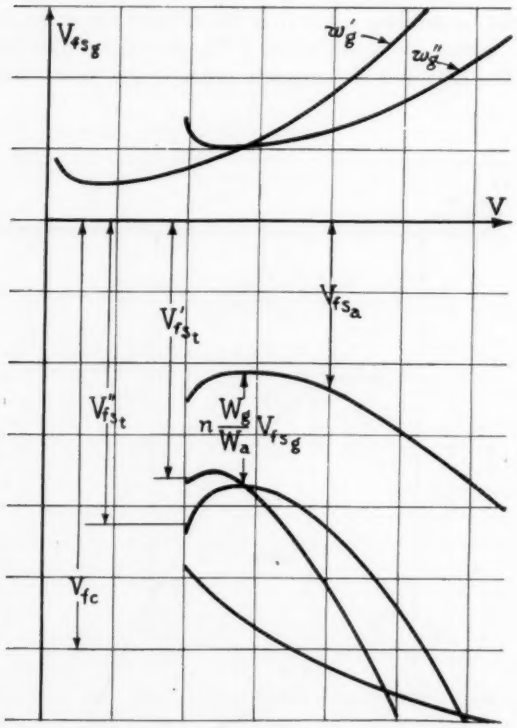

FIG. 3.

As shown in the lower portion of Fig. 3, the effect of an increased wing loading is to shift the combined sinking speed, V_{fs_t}, also upward and to the right. This leads to the conclusion that increasing the wing loading of the tow improved the conditions in the high-speed region. This conclusion, however, is subject to many limitations.

Cruising Speed

For the tow,

$$V_{fs_g} = \left(\frac{2w_g}{\rho C_{L_g}}\right)^{1/2}\left(\frac{C_{D_g}}{C_{L_g}}\right) = V\left(\frac{C_{D_g}}{C_{L_g}}\right) \qquad \text{(I-20)}$$

Imagine that an extremely small glider is attached to a fast, heavily loaded tug whose cruising speed is high and whose C_{L_a} is low at cruising. The question may be raised as to what the loading of the tow should be so as to affect the cruising speed as little as possible—that is, to make the sinking speed, V_{sf_g}, as small as possible for the given value of the forward speed

$$V = (2w_g/\rho C_{L_g})$$

It is evidently necessary to make C_{D_g}/C_{L_g} in equation (I-20) as low as possible. For this particular case, then, it is desirable to operate at best L/D, which gives a high C_{L_g} as compared with the relatively low C_{L_a}. With V constant, w_g will be somewhat higher than w_a, since

$$w_g = w_a(C_{L_g}/C_{L_a})$$

In this particular case, then, the performance would be improved by increasing the glider wing loading appreciably, in agreement with Fig. 3. Such a requirement may contradict other requirements for the tow such as low landing speed and freedom from the stall.

If, on the other hand, an extremely large glider or a large number of small gliders is attached to the tow, the air train will be operating at quite a low speed. Hence, the tug will be operating at a high angle of attack. But the tow should again operate at the best $(L/D)_g$ from reasoning analogous to the above. It is advantageous, therefore, to make the tow operate at a lower C_L and, again referring to Eq. (I-3), it is seen that w_g should in this case be lower than w_a.

The contradiction in the two conclusions shows how dangerous generalizations can be.

Rate of Climb

The best climb of the tug will occur somewhere in the neighborhood of the best power coefficient, $C_D/C_L^{3/2}$, which is at fairly high angle of attack. Whether in forward speed or in climb Eq. (I-20) still applies, and the fictitious sinking speed of the tow will be a minimum at best L/D. Hence, for least loss in climb we should put the tow at a lower angle of incidence and C_L. Again referring to Eq. (I-3), it can be concluded that here also the loading of the glider should be lower than that of the tug to get best climb performance.

Maximum Range

If the tug is operating at the attitude for best range—that is, at best L/D—it is clear that the tow should also operate at its best L/D. At best L/D,

$$C_{L_a}^2 = \pi R_{e_a} C_{D_{p_a}}$$

$$C_{L_g}^2 = \pi R_{e_g} C_{D_{p_g}} \qquad \text{(I-21)}$$

If R_{e_g} is appreciably greater than R_{e_a}, and $C_{D_{p_g}}$ is not much less than $C_{D_{p_a}}$, the tow should operate at a higher value of C_L than the tug and, hence, at a higher wing loading.

Power Required

The drag of an aircraft,

$$D = W/(L/D)$$

and the speed,

$$V = (2w/\rho C_L)^{1/2}$$

Hence, the horsepower required,

$$\text{hp.}_{\text{reqd.}} \sim DV \sim [W/(L/D)](2w/\rho C_L)^{1/2}$$

The following theorem can therefore be stated for the power required:

When the weight and aerodynamic proportions remain unchanged, the horsepower required and the speed vary in the same ratio with the wing loading at a constant value of the lift coefficient.

Power-Velocity Diagram

With the aid of the theorem just stated, the effect of change in glider wing loading can be readily investigated and is illustrated in Fig. 4. On the upper part of this diagram are drawn the curves of horsepower required and horsepower available for the tug; on the lower part of the diagram, the horsepower required for the tug for two different wing loadings. It is clear from these curves that if the high speed of the tug is to be affected as little as possible the horsepower required curve of the tow should be shifted to the right, so that the low power condition of the tow coincides with the maximum speed or high cruising speed of the tug. To decrease the speed of the tug as little as possible, it would seem desirable to increase the wing load of the gliders. Of course, this statement is only true within definite limitations.

For the best climb, on the other hand, it is desirable to make the minimum powers required coincide—that is, to decrease the wing loading.

The working out of practical examples is probably a better guide than these generalizations.

Aerodynamic Indifference to the Number of Gliders Towed

The horsepower required by the gliders in level flight is given by the expression $(nW_g/E_g)V$. It is clear from this expression that it is immaterial from an aerodynamic point of view whether n similar gliders of a combined weight nW_g are towed or a single similar glider having a gross weight equal to nW_g. The question whether a big glider or several smaller ones should be used is then to be settled with reference to possible gains in weight when smaller gliders are used, the greater personnel required for a number of gliders, piloting and group handling difficulties, and suitability for tactical purposes. It is also probable that if a number of relatively small gliders are employed greater aspect ratios will be permissible.

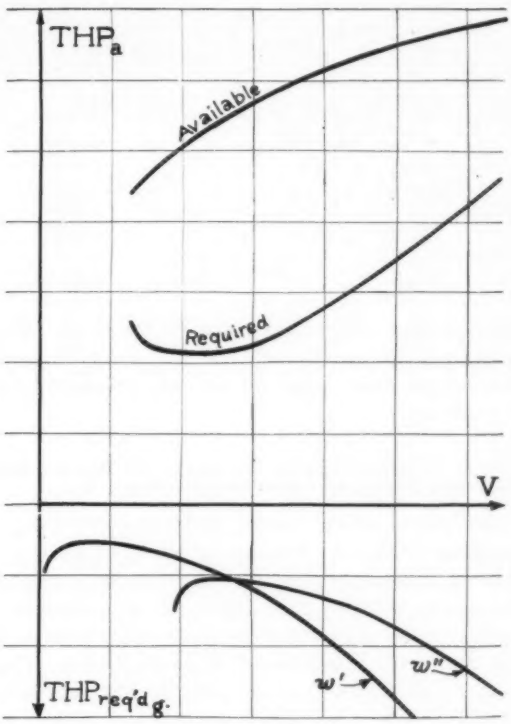

FIG. 4.

Aerodynamic Indifference Whether a Tug Alone or a Glider Train Is Employed

If the airplane alone were given so large an aspect ratio that its L/D was equivalent to that of the glider, there would be no aerodynamic advantage in the employment of a glider train. The possibilities of a glider train must be sought elsewhere than in pure aerodynamic considerations, though it is probable that high aerodynamic efficiency might be more readily obtained in a glider than in a powered airplane.

A Study of the Improvement in Induced Drag to the Greater Aspect Ratio of the Glider

Fig. 5 illustrates the induced drag of a train as a fraction of the induced drag of a powered airplane of the same weight as the train for varying ratios of glider weight over airplane weight. It is assumed that the wing loadings of the plane and glider are equal, and the

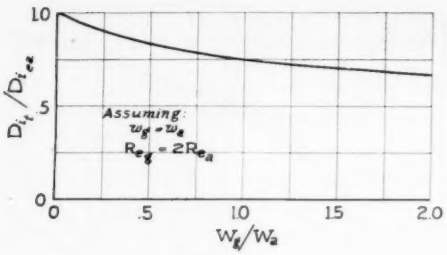

FIG. 5.

glider's aspect ratio is twice that of the plane. Since the wing loadings are the same, comparisons can be based on the same values for dynamic pressure, q, and lift coefficient.

Effect of Variation in Power Loading of the Tug Retaining Constant Horsepower and Wing Loading

Variation in power loading while retaining constant horsepower and wing loading, which is equivalent to variation in the size of the tug while retaining a constant horsepower, will now be studied. In all probability a decrease in size will involve no appreciable change in aspect ratio and the induced drag coefficient. There might be an increase in the parasite drag coefficient referred to wing area, since the size of the fuselage and engine nacelles would not decrease in the same ratio as the weight and wing area.

As an approximation, assume that the parasite drag coefficient remains a constant so that the value of L/D also remains a constant. It is also assumed that the aerodynamic characteristics and wing loading of the glider or gliders remain unchanged.

With the above simplifications it is apparent that V_{fs_a} and V_{fs_g} (dependent solely on wing loading and aerodynamic efficiency) remain unchanged and that the ratio nW_g/W_a increases. On the other hand, the fictitious climbing speed,

$$V_{fc} = 550\text{b.hp.}\eta/W_a$$

will increase, since the size of the tug has been diminished. In the equation of equilibrium,

$$V_{fc} = \frac{550\text{b.hp.}\eta}{W_a} = V_{fs_a} + (nW_g/W_a)V_{fs_g} \qquad \text{(I-18)}$$

it may be assumed that $V_{fs_a} > V_{fs_g}$, since the glider is, generally, more efficient than the tug.

If the change in weight of the tug is ΔW_a and the change in weight of the glider is ΔW_g, then with no change in the equilibrium speed,

$$\frac{550\text{b.hp.}\eta}{W_a + \Delta W_a} = V_{fs_a} + \frac{n(W_g + \Delta W_g)}{W_a + \Delta W_a}V_{fs_g} \qquad \text{(I-22)}$$

The problem is to find ΔW_g as a function of W_a and other related quantities so that there is no change in the equilibrium speed. Combining Eqs. (I-18) and (I-22)

$$W_aV_{fs_a} + nW_gV_{fs_g} = (W_a + \Delta W_a)V_{fs_a} + n(W_g + \Delta W_g)V_{fs_g}$$

from which $0 = \Delta W_aV_{fs_a} + n\Delta WV_{fs_{ga}}$:

$$n\Delta W_g = -\Delta W_a(V_{fs_g}/V_{fs_g}) \qquad \text{(I-23)}$$

From this equation it follows that:

(a) As the weight of the tug decreases, the weight of the glider increases when horsepower, wing loadings, and aerodynamic efficiencies remain constant.

(b) The greater the ratio, V_{fs_a}/V_{fs_g}, the higher is the increase in permissible weight of the glider for the same top speed and horsepower.

It is now logical to consider the pay load, assuming that,

$$W_{p_a} = K_{p_a}W_a$$
$$W_{p_g} = K_{p_g}W_g \qquad \text{(I-24)}$$

where

W_{p_a} = pay load of the airplane or tug
K_{p_a} = constant for the airplane
W_{p_g} = pay load of a glider or tow
K_{p_g} = constant for the glider

If $K_{p_g} > K_{p_a}$, as is generally the case, it is clear that the increase in pay load will be even more favorable than the increase in gross weight of the glider. Thus, from the point of view of the pay load, it would seem desirable to reduce the size of the tug as much as possible.

Effect of Change in Weight of the Tug on the Climb

By methods similar to the above, it is readily possible to study the effects of a decrease in tug weight on the climb when horsepower, wing loading, and aerodynamic efficiency remain unchanged, and the exact processes employed can be left to the ingenuity of the calculator.

Characteristics Required for a Good Tug

Certain tentative conclusions regarding the required characteristics of a good tug can be made on the basis of previous reasoning and equations. Thus, from Eq. (I-18), of equilibrium in level flight,

$$550\text{b.hp.}\eta/W_a = V_{fs_a} + (nW_g/W_a)V_{fs}$$

the following conclusions can be drawn:

(1) It is desirable, of course, to have both the tug and tow as aerodynamically efficient as possible in order to have the least possible sinking speed for a given air speed.

(2) It is desirable to have a low power loading W_a/b.hp. and therefore, by inference, a large excess horsepower so that a large glider weight can be towed without excessive loss of speed.

TABLE 1
Characteristics of Tug and Tow

	Tow Plane	Tow or Glider with Various Wing Loading-Aspect Ratio Combinations				
Wing loading, w/s—lbs. per sq.ft.	33.6	10		20		30
Aspect ratio, R	7.78	12	9	12	15	12
Gross weight, W—lbs.	18,500 (take-off)					
	17,500 (landing)	15,200	13,400	13,800	14,300	12,900
Weight empty, W_e—lbs.	12,000	8,700	6,900	7,300	7,800	6,400
Disposable load, W_d—lbs.	6,500	6,500	6,500	6,500	6,500	6,500
Disposable load, W_d—% W	35.2	42.8	48.5	47.1	45.5	50.4
Crew and equipment weight, W_c—lbs.	440	440	440	440	440	440
Wing area, S—sq.ft.	551	1,520	670	690	715	430
Span	65.5	135.0	77.8	91.0	103.6	71.8
Profile drag coefficient, C_{D_0}	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090
Nacelle drag coefficient $(C_{D_{par}})(S)$—sq.ft.	$0.0140 \times 551 = 7.71$	0	0	0	0	0
Equivalent parasite area (less nacelles)		7.71	7.71	7.71	7.71	7.71
Parasite drag coefficient, $C_{D_{par}}$	0.0210	0.0051	0.0115	0.0112	0.0107	0.0180
Profile plus parasite drag coefficient, C_{D_p}	0.0300	0.0141	0.0205	0.0202	0.0197	0.0270

(3) Since V_{fs_g} is likely to be less than V_{fs_a}, it seems desirable to have a relatively small tug. In particular, when top speed or high cruising speed is wanted, there should be advantage in reducing the weight of the tug and increasing the weight of the glider.

Referring to Eq. (I-17),

$$V_e = \left[\frac{550 \text{b.hp.} \eta}{W_a} - \left(V_{fs_a} + \left(\frac{nW_g}{W_a}\right) V_{fs_g}\right)\right] \frac{W_a}{W_a + nW_g}$$

conclusions 1 and 2 above still apply. However, if a reasonable rate of climb is to be maintained, care must be taken when increasing glider weight and decreasing tug weight. Even if the glider is more efficient aerodynamically so that the excess horsepower remains the same in spite of the increased glider weight, the rate of climb will diminish when the total weight of the train is increased.

All the above conclusions merit arithmetical investigation.

Desirable Size of Tug for a Given Wing Loading

Another problem that merits arithmetical investigation is the size of the tug to be employed. From the previous reasoning and equations it would appear desirable to have the smallest possible tug for a given engine power. However, the following are limitations on the possible decrease in size of tug:

(1) If the tug is given enormous horsepower for a small plane, the parasite drag coefficient is apt to increase unduly. For a single engine tug it is aerodynamically uneconomical to have a large engine in a small plane.

(2) If the tug is made extremely small for a given horsepower, the aspect ratio is likely to be small, and the aerodynamic efficiency suffers accordingly.

(3) Aileron control difficulties may appear when a short span is combined with a powerful engine torque.

(4) Longitudinal stability and control, power-on and power-off are apt to be seriously affected.

(5) If the tug is extremely small in relation to the size of the gliders, the danger may arise that the glider train may "control" the tug unduly.

Desirable Wing Loading for a Tug of Given Horsepower and Weight

The effect of change of loading on the horsepower required, with gross weight unchanged, is illustrated in Fig. 3.

It is clear from this diagram that if the wing loading of the tug is decreased below the conventional: (a) the excess horsepower available for climb will be increased and (b) the top speed will be diminished.

It is conceivable, therefore, that with a tug lightly loaded per square foot a much greater glider train could be towed than with heavy tug wing loading. It is probable, however, that not only would the tug cruising speed be low but the train cruising speed might become prohibitively low.

PART II—PERFORMANCE, RANGE AND PAY-LOAD CALCULATIONS FOR A GLIDER TRAIN

The theoretic considerations of Part I must of course be supplemented by calculations of performance, range, and pay load for a glider train which might be readily put into practice, and such calculations are presented in Part II of this paper.

Characteristics of Tug and Tow

The characteristics of the tug (approximating the characteristics of the Lockheed Lodestar) and of the tow or glider are given in Table 1. It will be noted that various wing loadings for the glider have been considered, with appropriate minor variations in weight and dimensions.

The hypothetic tug is assumed to be powered with engines with the following performance for each engine: b.hp. = 1,200 for take-off, = 1,050, normal rating, = 700, cruising.

It is of course possible, with a given tug, to imagine a large variety of glider train combinations. Thus the tug could conceivably be followed by one large glider. In this case flexibility of operation would be lost, dimensions might become cumbersome, and there might be difficulty in take-off. With three gliders, take-off with the use of separate tugs is conceivable. The glider finally selected for experimental computations was assumed to have a disposable load of approximately the same amount as the tug—namely, 6,500 lbs. It was also assumed that the parasite drag area of the glider would be the same as the parasite drag area of the tug less nacelles. The glider was given an aspect ratio of 12, which is in accord with practice of the day. In estimating the weight of the glider the tug was used as a basis of computation, and in arriving at the weight, empty of the glider, allowances were made for elimination of the power plant and nacelles; reduction in weight of landing gear; reduction in weight of instruments and equipment; and modification of weight of the wing because of changes in span, aspect ratio, and loading per square foot. While all these estimates are approximate, they are sufficiently accurate for the purpose of this analysis.

Horsepower Required and Varying Wing Loading

The horsepower required was calculated for $w_g = 10$, 20, and 30, and results were plotted against speed in Fig. 6. Since the flaps-up stalling speed of the Lockheed Lodestar is 93 m.p.h., the minimum level flight speed of the glider train was arbitrarily set at 115 m.p.h. as providing a sufficient margin above stalling. From Fig. 6 it is seen that increase in loading for the range of conditions investigated improves performance. Nevertheless, since landing speed is important in glider operation, w_g was arbitrarily chosen to be 20 lbs. per sq.ft. for further calculations.

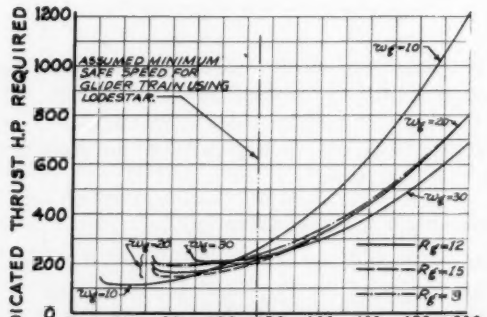

FIG. 6. Effect of wing loading and aspect ratio on glider power required. Disposable load maintained at 6,500 lbs.

Effect of Varying Aspect Ratio

With $w_g = 20$, the aspect ratio was systematically varied, and the horsepower required was calculated. As indicated by a curve in Fig. 6, the optimum value (taking weight changes as well as induced drag changes into account) was found to be $R = 12$. An aspect ratio of 12, accordingly, was the one used in further calculations.

Performance Curves with Varying Number of Tows

The power required for the tug alone and for the tug with one, two, three, and four gliders in tow (with $w_g = 20$, $R = 12$ for every glider) is shown in Fig. 7. The power required is given in terms of indicated power where

$$hp_{\cdot reqd.\ indicated} = hp_{\cdot reqd.}\ (\sigma^{1/s})$$

$$V_i = V(\sigma^{1/s})$$

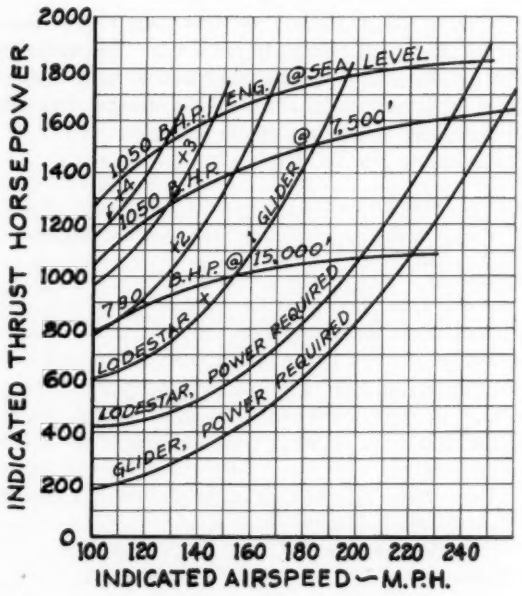

FIG. 7. Thrust horsepower versus air speed. Lodestar weight = 18,500 lbs.; glider weight = 13,800 lbs.

The available or thrust horsepower is also given in the form of indicated horsepower for sea level, for critical altitude of 7,500 ft., and for 15,000 ft. Ram was added to each altitude, and the rate of climb was corrected for the acceleration that occurs when constant indicated climbing speed is maintained.

The results of these calculations, rate of climb, and true air speed at different altitudes are summarized in Fig. 8.

Range

Because the usefulness and efficiency of any air transport unit depends on the factors of pay load, range, air speed, and power required, the subject of range has

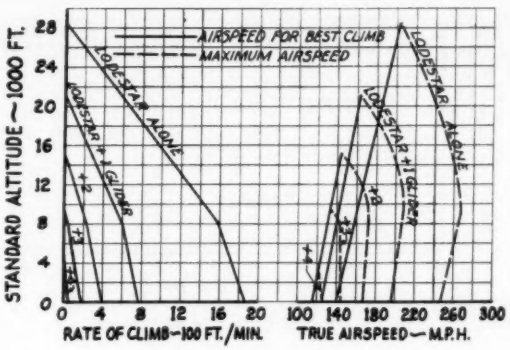

FIG. 8. Performance summary. Lodestar weight = 18,500 lbs.; glider weight = 13,800 lbs.

been studied. First, the maximum range at 5,000 ft. has been determined as a function of Lodestar weight, with the results shown in Fig. 9. At each of several weights, at the speed for maximum L/D, and at the engine r.p.m. for maximum η/c, the fuel consumption was found from which the range was calculated.

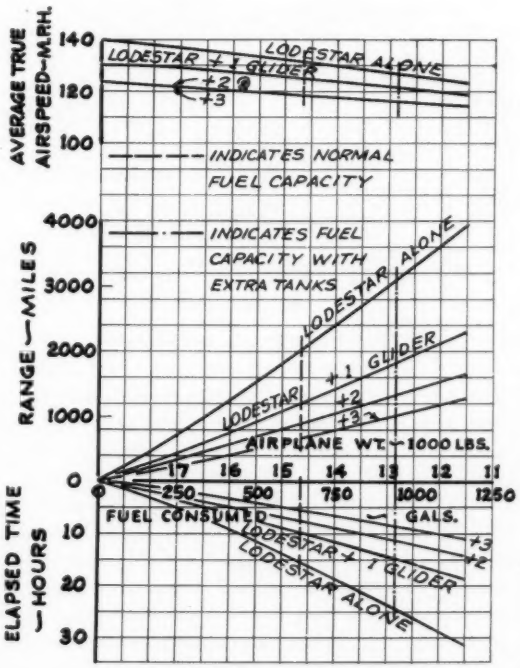

FIG. 9. Maximum range flight conditions. Altitude = 5,000 ft.

Next, the range was found with the Lodestar operating at a constant cruising power of 700 b.hp. per engine. The results of this investigation are shown in Fig. 10.

Payload-Range Analysis

For the Lodestar, the disposable load

$$W_{d_a} = W_a - W_{e_a} = W_{p_a} + W_{f_a} + W_{c_a}$$

where

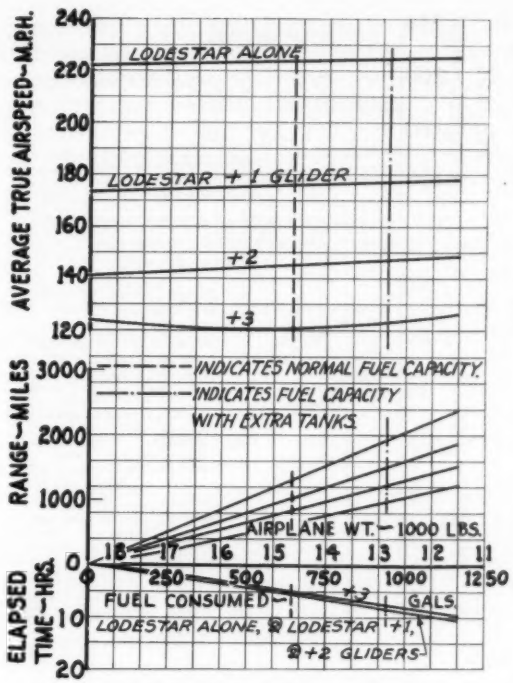

FIG. 10. Cruising range flight conditions. 700 b.hp. per engine at 2,325 r.p.m.; altitude = 5,000 ft.

W_a = gross weight = 18,500 lbs.
W_{e_a} = empty weight = 12,000 lbs.
W_{c_a} = crew of two with equipment = 440 lbs.
W_{f_a} = fuel and oil load
W_{p_a} = pay load

Then the pay load,

$$\begin{aligned} W_{p_a} &= W_a - W_{e_a} - W_{c_a} - W_{f_a} \\ &= 18{,}500 - 12{,}000 - 400 - W_{f_a} \\ &= 6{,}060 - W_{f_a} \end{aligned}$$

In this investigation it was assumed that the weight of the oil carried is one-twenty-fourth the weight of the fuel and that 10 per cent surplus fuel is carried for a flight of a given range. Reading the range for a given amount of fuel consumed from Figs. 9 or 10, the pay load for this condition becomes

$$W_{p_a} = 6{,}060 - 1.1 W_{f_a}$$

When flying a distance requiring more fuel than the Lodestar's normal tank capacity, 644 gal. = 3,860 lbs., extra tanks are placed in the Lodestar. These extra tanks are assumed to weigh $^1/_2$ lb. per gal. or 1 lb. for each 12 lbs. of gasoline. The pay load then becomes

$$W_{f_a} = 6{,}060 - [W_{f_a} + 0.0833\ (W_{f_a} - 3{,}860)]1.1$$

Since the take-off weight is 18,500 lbs. and landing gross weight is 17,500 lbs., the maximum theoretic pay load is 5,060 lbs. (i.e., with no fuel load).

For each glider, the disposable load,

$$W_{d_g} = W_g - W_{e_g} = W_{p_g} + W_{c_g}$$

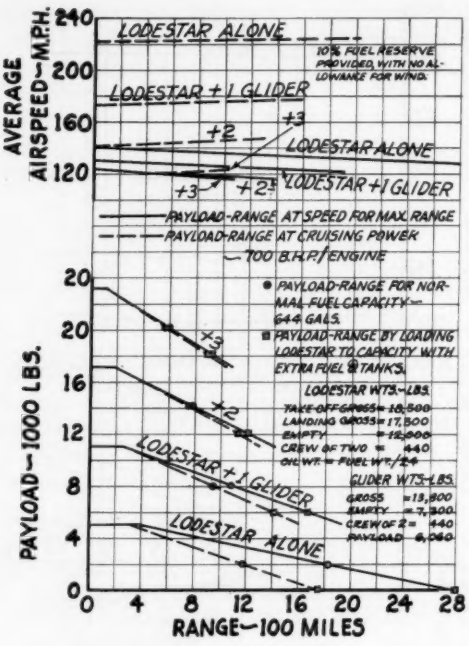

FIG. 11. Payload-range analysis.

where

W_g = gross weight = 13,800 lbs.
W_{e_g} = empty weight = 7,300 lbs.
W_{c_g} = crew of two with equipment = 440 lbs.
W_{p_g} = pay load

Then the pay load,

$$W_{p_g} = W_g - W_{e_g} - W_{c_g} = 13{,}800 - 7{,}300 - 440 = 6{,}060 \text{ lbs.}$$

For a number of values of fuel consumed the pay load is computed, and the range is read from Fig. 9 or Fig. 10, to give the payload-range analysis curves, Fig. 11.

Ratios for Comparison of the Tug with the Glider Train

Three ratios of comparison between the tug and the glider train have been devised, and the curves illustrating these ratios are illustrated in Figs. 12, 13, and 14.

(1) Pay-Load Factor

The pay-load factor, G.T.P.F. = pay load$_{train}$/pay load$_{tug}$ for a given range. In Fig. 12 this ratio is computed under conditions of flight for best range (as previously defined). It is under the conditions of best range that the pay-load factor attains the higher value.

(2) Advantage Factor

The advantage factor G.T.A.F. = (pay load × air speed)$_{train}$/(pay load × air speed)$_{tug}$ for a given range. In Fig. 14 this ratio is computed for conditions of cruising speed (as previously defined) because it is under cruising condition that the advantage factor attains the higher value.

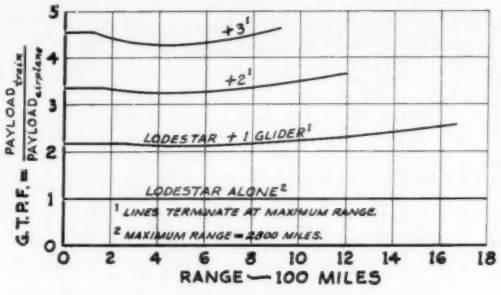

FIG. 12. Glider train pay-load factor.

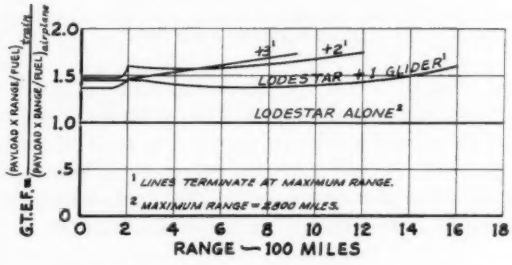

FIG. 13. Glider train efficiency factor.

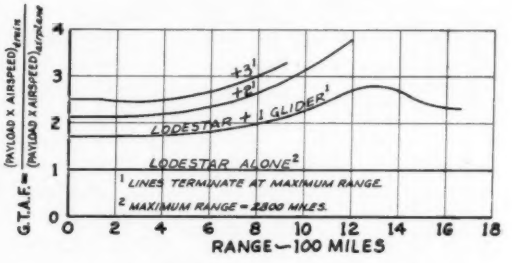

FIG. 14. Glider train advantage factor.

(3) Efficiency Factor

The efficiency factor, G.T.E.F. = $\{[(\text{pay load} \times \text{range})/\text{consumed fuel}]_{train}\}/\{[(\text{pay load} \times \text{range})/\text{consumed fuel}]_{tug}\}$ is computed for the condition of best range, which gives the higher value.

It will be noted that in the computation of all the above factors one of the two conditions of operation is selected which gives the higher value to the efficiency factor.

For instance, the maximum pay load or (pay load times range) is obtained by flying at the speed for best range, while maximum (pay load times air speed) is obtained when flying at cruising speed except when the range desired is greater than the maximum cruising range.

Some of the curves show irregularities, the reasons for which may not be immediately apparent to the reader. Referring to Figs. 10 and 11, it is seen that the average air speed for a train with three gliders decreases with

range, contrary to the trend of the other combinations. In the calculations the minimum safe flying speed of the Lodestar was assumed to vary uniformly from 115 m.p.h. for 18,500 lbs. to 100 m.p.h. for 12,500 lbs. Since more than the 700 cruising b.hp. per engine is required by a three-glider train at the beginning of flight to maintain a safe flying speed, the air speed decreases until the Lodestar becomes light enough that normal cruising power will be sufficient to give a safe speed.

Referring to Figs. 12, 13, and 14, the glider train pay-load factor remains constant for those ranges where both the train pay load and the Lodestar pay load are constant. Referring to Fig. 11, the pay load of the train starts decreasing sooner than the pay load of the Lodestar, causing the pay-load factor to decrease until the Lodestar pay load also decreases. This also causes irregularities in the glider train advantage factor and the glider train efficiency factor.

The irregularity in the glider train advantage factor for one glider is explained by the fact that the Lodestar shifts from the cruising range to the maximum range condition at 1,300 miles while the one-glider train does not shift until its maximum cruising range of 1,400 miles is reached. The difference in slope of (pay load times air speed) versus range for maximum range and cruising range causes the irregularity.

At the beginning of range flight with two and three gliders, sufficient power is required so that it is necessary to use an automatic rich mixture, causing a jump in the glider train efficiency factor for these two trains when the switch is made to auto lean.

Part III—Discussion of Results of Calculations

No amount of theoretic discussion will determine the possibilities of the glider train. Coupled with theoretic discussion, there will have to be service experience and, coupled with service experience, careful analysis of operating costs. Nevertheless, discussion of the calculations in Part II leads to some tentative conclusions:

(1) The weight/take-off horsepower of the Lodestar is 18,500/2,400 = 7.7, and its maximum excess horsepower at sea level and normal rating is approximately 1,000 hp. The ratio $3W_g/W_a = [3(13,800)]/18,500 = 2.23$. If a tug is to be able to tow gliders whose weight is appreciably in excess of its own weight, there must be low power loading of the tug as well as good aerodynamic efficiency of tug and tow.

(2) Without departure from conventional design, a glider will have much higher aerodynamic efficiency than even an excellent powered airplane. Hence, from the point of view of aerodynamic efficiency, the tow should fly high above the tug, thus transferring load to the wings of the tug. Such arrangement might be dangerous because it might lead to overriding and excessive stresses in the cable.

(3) The calculations indicate one serious difficulty for the glider train. When the weight of the tow is large in relation to the weight of the tug, reasonable range can be secured only by combining quite low cruising speed with appreciable power drawn from the engine. This means that cooling difficulties are certain to occur. Either liquid-cooled engines or blower cooling for air-cooled engines would become necessary.

(4) When the tug is towing three gliders, rate of climb at sea level and the ceiling become inordinately low. It is conceivable that some method of assisting the take-off could be developed, but the glider train could only operate satisfactorily over low lying country.

(5) With normal fuel capacity range becomes severely restricted.

(6) On the other hand, pay load increases enormously.

(7) It is the enormous increase in pay load which explains the fact that both the advantage factor (pay load $\times$ air speed)$_{\text{train}}$/(pay load $\times$ air speed)$_{\text{airplane}}$ and the glider train efficiency factor $\{[(\text{pay load} \times \text{range})/\text{consumed fuel}]_{\text{train}}\}/\{[(\text{pay load} \times \text{range})/\text{consumed fuel}]_{\text{airplane}}\}$ appear so advantageous.

(8) Enthusiastic approval of the glider is generally based on the increased pay load and on the improvement in such efficiency factors as the two given above. Such advantage cannot be taken at face value for ordinary air transport operation. Such ratios merely indicate that for a low horsepower loading a high horsepower loading has been substituted. If actual costs of operation are examined, it will be found that several factors militate against the apparent advantage. Because of the difficulties of the take-off and the low rate of climb and the short range, the block-to-block speed will be more sharply decreased in relation to the cruising speed, which is the case in normal air transport operation. With head winds the block-to-block speed would become quite unsatisfactory. Glider train efficiency factor would be greatly decreased because of the extra gasoline consumed in take-off and climb, and the relatively short-range flights that would be permissible in the glider, when reserve fuel is properly taken into account. (However the disadvantages in take-off, climb, and range of glider trains might be partially overcome by refueling the tug in flight and by towing additions to the train at intermediate points to the flight altitude by an auxiliary tug, where they could be coupled in flight.)

(9) In estimating direct flying costs almost every item of cost per mile (pilot's pay, insurance, maintenance, depreciation, etc.) varies inversely as the block speed. With the block speed sharply reduced for the reasons mentioned previously, it is not at all certain that the direct cost per mile would be lowered. On the contrary, the direct cost per mile might actually go up, as it is apt to do when an ordinary transport airplane is designed with a high pounds per horsepower loading. (Estimates of direct flying costs are now being under-

taken by the authors and will be submitted in an appendix to the present paper.)

(10) In setting forth the merits of the glider train, its proponents should not fall into the error of some friends of the helicopter who, in addition to claiming the advantages that the helicopter indeed possesses—such as extraordinary ability to operate in restricted terrain and its power of hovering for close observation, its serviceability for short hop work—also announce that the helicopter will ultimately develop great speed and efficiency (which is not at all likely). The proponents of the glider train would do far better to regard it quite realistically and to emphasize a number of services which the glider train can really perform.

(11) In a military situation, for example, where large numbers of troops or quantities of supplies are to be transported over a short stretch of sea or over a short stretch of level land, the enormous cargo carrying capacity of the cargo train would be invaluable.

(12) In air transport operation, a glider pickup service would be invaluable and would be a logical corollary to the mail pickup service, which has already proved so valuable. The calculations indicate that where the tow picks up but a single glider, climb (and hence take-off) will be entirely satisfactory.

(13) Again, in air-cargo operation, the enormous increase in pay load possible by the use of a glider train would give the air-line operator highly useful and inexpensive overload or emergency equipment.

(14) Finally, the calculations indicate that the possibilities of the glider train cannot be fully exploited with the use of the airplane and power plant as they are available at the present time, but that it would be well worth while to design an aerial locomotive or tug truly adapted to such service. The general specification would be somewhat on the following lines:

(a) The power plant would be so designed that adequate cooling would be available at a relatively slow speed.

(b) The power plant would be so designed that by means of change speed gear, variable pitch, variable compression ratio or similar means the useful working range of the engine would be extended from full power to a small fraction of the full power.

(c) When traffic is light, the tug would operate alone at high speeds and at a fraction of full engine power. Hence, the tug would be small and highly "overpowered."

(d) As traffic grew heavier, the operator could progressively add glider after glider to his train, the flight engineer would be able to increase the power of the engine to an almost corresponding degree, and the train would function without too great a loss of speed.

(e) By a combination of such means a truly, flexible, economically worth while piece of equipment might be placed in the hands of the air line.

(f) Since it would be advantageous to run the tow cable from the center of gravity of the glider to the center of gravity of the airplane, designers might perhaps care to give consideration to a tug airplane with tail surfaces first—i.e., to the Canard type.

Limits of Human Heat Regulation

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Introduction

THE HUMAN BEING is in possession of a complicated and beautifully integrated mechanism that holds his internal temperature at a nearly constant level in spite of the varying production of internal heat and the wide range of ambient temperatures to which he is exposed. We are concerned with the limits of this mechanism. When the air temperatures are at desert levels, it may be impossible for the individual to lose heat at the rate at which he is producing it. This is especially the case when the relative humidity is also high. The increasing heat load causes a number of manifestations varying from lessening efficiency to sudden collapse, which may be fatal. In contrast, prolonged external cold may cause loss of heat at a rate greater than the production of heat. The consequent cooling of the extremities in the early states interferes with the movements of the hands and feet. Continued exposure causes great discomfort and may result in serious injury to the hands and feet. Equally important is the prolonged fatigue that results from the cooling of the internal organs.

Kinetic Energy

The animal organism obtains all of its kinetic energy by conversion of the potential energy contained in its food. All of this kinetic energy manifests itself as heat within the body except for a small and variable amount that is converted to external work. Since the organism is never more than 25 per cent efficient, generally less and often very much less, most of the energy involved in the performance of external work is manifested as heat. Under comfortable conditions the individual loses heat from the surface of the body at the rate at which it is produced internally. This is accomplished by means of radiation, conduction, convection, and evaporation of water. The proportion carried away by each process depends upon the external conditions over which the organism has no control and the changes in the temperature of the skin. This surface temperature, so far as the organism is concerned, is related to the volume of blood that flows through the integumentary layers of the body. When the ambient and skin temperatures are equal, the only means of losing heat is vaporization of water. Cold external conditions cause the blood to be kept away from the skin, whose temperature falls and so reduces loss of heat. The opposite events take place in hot atmospheres. Each of these mechanisms has definite limits. When the external conditions are such that the loss of heat is either greater than or less than the heat production, abnormal manifestations develop and become increasingly severe. If not interrupted, they end in death.

Internal and Skin Temperatures

The individual is comfortable when the internal temperature is 99°F. and the average skin temperature is 93°F. The fasting resting nude man will maintain these temperatures while the ambient temperature is 86°F. and air movement is minimal. This interval of 7° between skin and air temperatures establishes a rate of heat loss equal to its production under these quiet conditions. Lessening of the air temperature several degrees causes fall of skin temperatures, and this seems to be the stimulus to increased heat production of moderate degree, which for a time again equals the loss of heat. Still cooler external temperatures often cause shivering which results in a heat production that is two or three times as great as that of the quiet relaxed state and again compensates for the greater loss of heat. Strenuous exercise is capable of increasing the heat production eight- or tenfold while it lasts, but it is too exhausting to be continued long at this rate. Thereafter the organism exposed to a cold environment will lose heat more rapidly than it is producing it, as evidenced by a progressive fall of internal temperature.

Clothing

Clothing, as everyone knows, is capable of lessening the effect of external cold, and its value is conveniently expressed in terms of Clo units. One such unit is defined as the amount of insulation needed to maintain the skin temperature of the quiet individual at 93°F. when the ambient temperature is 70°F. and the air is still. It corresponds roughly to the usual clothing worn by men in warm weather. Since the naked subject is comfortable when the air temperature is 86°F., one Clo compensates for a lowering of ambient temperature by 16°F. (86° − 70°). Theoretically, then, when the air temperature is zero, 5 Clo will be required to maintain the skin temperature of an inactive man at a comfortable level. Rapid movement of air will make it necessary to increase the thermal insulation. Vigorous exercise, on the other hand, will lessen the needed

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insulation in proportion to extra heat produced. However, a significant increase in activity is impossible on the part of those aviators who are in the outlying stations because of their cramped positions, and they are the very ones who need the extra heat most.

As pointed out earlier, duration of exposure to cold air is at least as important as actual temperature. Thus it is a fact that clothing that has an insulating value of 4 Clo and, hence, will protect theoretically against air temperatures of 22°F. fails to do so in actual flights when the ambient temperature is 30°F. if the aviator is in the air more than six hours.

Experiments

The effect of duration of cold was clearly demonstrated by exposing a subject to a series of temperatures for many consecutive hours. He wore the warmest assembly of regulation unheated flying clothing available at the time. It was found that even ambient temperatures as high as 32°F. caused a slow fall in skin temperatures. However, moderate falls up to 5° were not objectionable since they did not cause real discomfort. However, further falls are accompanied by increasing discomfort. When the temperature of the skin over the trunk has dropped to 80°F. and that of the toes is down to 40°F., the subject experiences extreme discomfort, is no longer efficient, and may suffer anatomic injury of the extremities. Table 1 shows how long it took the skin over the trunk and toes, respectively, to reach these dangerously low temperatures.

Table 1

Air Temperature	Duration in Hours	
°F.	Trunk	Toe
30	20	7
15	10	5
0	2.7	4.5
−15	3.0	3.5
−30	2.7	3.2
−45	2.3	2.8

Another investigator has measured the loss of stored bodily heat under similar circumstances. He found it to be 74 kilocalories per hour in excess of the heat produced when the ambient temperature was −31°F. The heat production in this case was 128 kilocalories per hour. This is an increase of about 50 per cent above what it would have been had the man been comfortable. In order to have prevented the progressive cooling off of his body while exposed to the low temperature he would have had to produce 200 kilocalories per hour. Since the organism is fully capable of producing heat at this rate, it is baffling to find that it does not respond adequately to the stimulus of cold. In fact, other observers have been unable to record any significant increase in heat production by pilots clothed in heavy winter flying suits even when they were undergoing a progressive loss of stored heat.

Protection Against Cold

Since the aviator does not produce sufficient heat to protect himself against prolonged cold, other means of preventing the harmful effects of cold are being sought. They are: (1) heat supplied to the plane itself. This is an engineering problem and, hence, will not be discussed by the writers. (2) Heat supplied to the clothing by electrical resistance. In the laboratory this method undoubtedly succeeds. In practice it is objectionable because it is a serious drain upon the available current and because the pilots have no confidence in it. They fear burns and they anticipate that an enemy bullet will cut the supply line. (3) Increase in insulating value of clothing. This cannot be accomplished by increasing the thickness of fabrics now in use, since the flying clothing is already so heavy and bulky that it lessens agility and increases the hazard of drowning. Success here depends upon the development of a fabric whose capacity to retard heat flow is at least twice that of the textiles now in use for equal thickness and weight.

High Air Temperatures

High air temperatures may cause as much disturbance of the individual's heat balance as cold ones. When the ambient temperature is as high as or higher than the skin temperature, vaporization of water is the only method of ridding the body of the heat it is producing. Under these circumstances a man doing hard work will produce a liter of sweat per hour, sometimes more. If all of it is evaporated, 580 kilocalories will be lost from the body. The dry air of the desert will permit the vaporization of all of this water. Loss of heat at this rate is ample to keep up with the heat production of prolonged work. However, the individual may also be absorbing heat from his hot surroundings and thus have a much greater total amount of heat to dissipate in order to prevent accumulation of body heat. It has been calculated from laboratory data that the thoroughly trained and acclimatized soldier can engage in the usual noncombatant field activities without accumulating a dangerous heat load for eight hours daily at air temperatures as high as 125°F., provided the relative humidity is not above 20 per cent. The removal of the heat involved will require the production of 6 or more liters of sweat and it is this loss that may cause serious disturbances. A few words will explain why this is so. The human adult body contains about 45 liters of liquid of which 30 is within the cells and 15 is outside the cells. This extracellular fluid, which includes the liquid portion of the blood, is characterized by the constancy of its composition. It contains 8 Gm. of sodium chloride per liter of water. Sweat is derived from the extracellular fluid and resembles it in composition but is more dilute. That produced by acclimatized persons contains about 2 Gm. of sodium chloride per liter. Accordingly, sweating removes four times as much water as salt from the extracellular

fluid and causes it to become abnormally concentrated. This is compensated for by a flow of water from the cells and by drinking water. When the latter is not taken in sufficient amounts, the loss of water from the cells causes the well-known disturbances—heat stroke and heat exhaustion—which, if not quickly relieved, end in death.

Even if the water lost as sweat is adequately replaced, it will remain within the body only in amounts that re-establish isotonicity. Since sodium chloride has also been lost, the total volume of extracellular fluid will be proportional to the amount of sodium chloride remaining in the extracellular spaces. It has been established that the unreplaced loss of 4 liters of extracellular fluid will initiate serious disturbances. Continued losses cause profound weakness, unconsciousness, and death. Since practically all of the sodium chloride in the body is in the extracellular fluid, it can be replaced only from the outside. If the usual food is eaten, 10 to 15 Gm. of salt will be returned daily. But twice this amount may be lost every day, and the subject does not become aware that this is taking place. Someone who understands the situation must be responsible for replacement in sufficient amounts.

Humidity

When the individual must contend with high humidity in addition to heat, he is at a further disadvantage, for now vaporization of water is retarded. He still sweats as profusely but some of it runs off and this portion removes no heat. The rate at which he can lose heat is now limited by the moisture content of the air.

In the Tropics where the relative humidity is in the eighties, even the acclimatized person engaged in the usual field activity cannot maintain heat balance unless the ambient temperature is less than 90°F. so that some heat can be lost by methods other than vaporization. Since the tropical nights are almost as warm and more humid than the days, loss of accumulated heat is a slow process. Consequently, the individual may start the next day with a positive heat balance. Under such circumstances he will soon feel exhausted and be unable to work hard. If he is driven he may collapse almost without warning. Accordingly, it is of the greatest importance to avoid storage of extra heat in the Tropics by appropriate restriction of physical work.

Letter to the Editor

Dear Sir:

Flax's article about three-dimensional flutter analysis[1] appears to have a striking similarity to a paper by the writer appearing in 1929 on the same subject,[2] which is practically unknown and rarely mentioned in the literature. This may not appear at the first sight, since the writer's paper lacks all the mathematical and methodical elegance of Flax's article, being written at a time when he was engineering draftsman for a purely commercial aircraft company 15 years ago and when the airplane's speed had just increased sufficiently to herald the undesired appearance of wing flutter. It was based on familiar engineering conceptions and stated that irrespective of individual distributions of mass and air loads over the span the "end conditions" of the elastic deflection curve together with the conditions of vanishing of all forces and moments for a free oscillating body (controlling the nodal points) determined the amplitude distribution to such an extent that it was permissible to consider only one prevalent amplitude distribution for each "degree of freedom." Thus it was concluded that only three linear differential equations representing wing bending and torsion and aileron motion need to be considered in practical applications.

In familiar engineering fashion, these equations were written in the form of summation of all moments from mass, air, and elastic forces at the wing root equivalent to Flax's equation.[3] Somewhat crude attempts were made to consider also the effect of coulomb dry friction and hysteresis damping, finite span and wing thickness, and stiffness of controls. Air forces for oscillating motion being known at that time only for small values of "reduced frequencies," interpolation formulas for higher values were set up and it is in this respect that greatest progress in the art was made since those "good old days."

The results were applied to two of the company's airplanes which had shown flutter and good agreement between predicted and tested flutter speeds and modes were arrived at.

For those interested in the development of the art, reading of the writer's rather condensed article[2] may still be worth while.

Felix Nagel
St. Louis, Mo.

[1] Flax, Alexander H., *Three-Dimensional Wing Flutter Analysis*, Journal of the Aeronautical Sciences, Vol. 10, No. 2, p. 41, February, 1943.

[2] Nagel, Felix, *Wing Flutter in Stationary Air Flow*, Luftfahrtforschung, Volume 3, May, 1929. (See also translation No. 285, Matériel Division, U.S. Air Forces, Dayton, O., and Journal of the Aeronautical Sciences, Vol. 7, No. 6, p. 256, April, 1940.)

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[1]Englund, C. R., Crawford, A. B., and Mumford, W. W., *Some Results of a Study of Ultra-Short-Wave Transmission Phenomenon*, Proc. I. R. E., Vol. 20, No. 12, pp. 481 and 482, March, 1933. Please give author, title, edition, volume, page, publisher and date of publication as indicated. Omission of one required fact causes much extra editorial work and possible inaccuracies. All references are grouped at the end of the article.

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